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HANDBOOK OF GEOGRAPHY

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HANDBOOK OF GEOGRAPHY

DESCRIPTIVE & MATHEMATICAL

BY

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IN TWO VOLUMES

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PREFACE

In all his works and lectures the author has always dwelt on the immense importance of geographical factors both on past history and on present life. What he has ventured to call geopolitics, or the combined influence of geographical with political facts, is one of the most decisive elements in human institutions. Having finished the first part of his historical works proper, together with two elaborate historic atlases on a new graphic plan, the author felt that a new handbook of Geography might be published in which the physiographic or descriptive part is given due eminence, and by means of which the reader of history might acquire a lifelike view of the actual physiognomy of a given country. The extensive travels of the author seemed to authorise him to undertake such a task. However, he was aware that it was impossible to do justice to the idea of a readable and lifelike handbook of geography without availing oneself of the material scattered in a thousand excellent books of travel or geographical research. Accordingly the author has freely helped himself to what in his judgment were the best sources of geographical knowledge, much as has been done by all writers in general geography, from the great Karl Ritter and Reclus downwards. To name every one of them would have been an unnecessary increase of the bulk of the work. The sources used are in twelve languages, and constitute the common property of all

geographers. In very numerous places the author has discarded or modified the statements of his authorities, on the strength of his own observations.

The first part, or descriptive geography, treats of the various countries of the five Continents, and the chief aim was to enable the reader to form a fair image of each bigger landscape; or, in other words, to view each country, or big sections thereof, from a standpoint so high in air as to admit of taking in at a glance entire provinces. As an inevitable consequence of the predominance of the descriptive standpoint, the statistics contained in the work may not in every single case be up to date. For a variety of reasons, the work took a large number of years to reach its completion, and it was therefore impossible to reconcile, in every item, both statistical and descriptive facts. Yet even this will, it is confidently hoped, not be found an obstacle at all, the principal point and object of the work being to impart knowledge of geographical physiognomies.

The second or mathematical part was added because the author had, at the inception of the present work, found practically no book in English on the subject of mathematical geography. In fact, at the office of one of the largest map-making establishments in England, the clerks said they had never heard of the term "mathematical geography." In this second part the attempt is made to show, from the best available sources, first how we succeeded in determining correctly the geometrical situation of a given town or hill on the globe; and secondly, how we succeed in locating this situation on a given map. Space, far from being a mere abstract concept, is, together with what it contains, the basis of all human activities: it is not only its *locus*, but also its foster-earth and cause. The accurate survey of geographical space, the precise determination of each locality within global space, and the correct reproduction of that global space on a plane such as is a map, *i.e.* a piece of flat paper—all this is part and

parcel of sound geographical knowledge. The author may state that he has always paid close attention to the history and technique of mathematical geography, and on compiling the present essay he has drawn only on such sources as he has, from personal experience of study and instruction, found really useful in the comprehension of the subject. The author will be happy to reply to any difficulty encountered in the study of mathematical geography on the part of his readers. He hopes that the studious public will gain some advantages from the present handbook which are not easily to be met elsewhere.

EMIL REICH.

ST. LUKE'S ROAD, LONDON, W.,
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INTRODUCTION

THE study of geography has been afflicted with numerous drawbacks and retarded by many misunderstandings. It is easy to explain what is meant by physics or chemistry; it is not quite easy to show in a few words what is meant by geography. To the indifferent, geography is a sort of elaborate gazetteer; to the learned students of Nature, on the other hand, it is the natural description of the earth, or what, with a more ambitious term, we might call terrestrial physiography. Yet other classes of scholars, such as historians or economists, consider geography as the handmaid and appendix of a promiscuous bevy of historical or politico-economical questions; while mathematicians are inclined to reduce it to a mere case in astronomy.

As with the theory or science of geography, so it is with the practical work of geographical maps or charts. It is not long since that the most advanced nations of Europe have given serious thought to the science and art of map-making, and it is only within our generation that great and systematic attention has been paid to the true representation of the surface or morphology of countries. To the present day mountain ranges are but too frequently designed as rampant caterpillars, and the course of rivers is more in accordance with the erratic fancy of the cartographer than with Nature's own arrangements. Yet by every step in the study of geography we are warned, nay, forced, to attach ever greater importance to the construction of true maps; and the time will come when geographical maps will be generally known to be of the same significance

and power as are mathematical formulæ, maps being in reality nothing but graphical formulæ of countries, lakes, rivers, etc.

This curiously inarticulate and floating character of geography is owing mainly to the fact that, unlike students of most other branches of science, the geographer is sitting astride on more than two fences, and is thus referring to more than one field of investigation. In that he resembles frontier countries where various nations are meeting, such as Austria; and the theoretic *Ausgleich* between the conflicting claims of the various elements of geography is also one beset with great difficulties. For geography refers to the earth as well as to man. It is, on the one hand, sunk in the mineral life of the inorganic elements of the earth; and, on the other, it reaches through the vegetable and animal world into some of the innermost recesses of the highest organic manifestations of man. To these two modes of life even a third, the cosmic life of the earth, must be added, which no geographer can quite ignore, although he must not take upon himself all the abstract glory of astronomy. At different times, one or the other of these three factors of geography was made a preferential study of, and it is only in our comprehensive time, when most things tend to internationalisation and common activity, that the various elements of geographical study are beginning to interpenetrate one another.

In geography the earth is always considered in the first place as the habitat of man. Such problems of geophysics, as the natural history of the earth has aptly been called, as do not in any known fashion act or react upon man, are not a proper object of geography. Man, it is true, is so intimately allied with or arrayed against the earth, that it is impossible, in general terms, to set bounds to the incorporation of geophysics within the precincts of geography proper. Thus the study of the earth's interior may, through the reaction of that interior against the surface of the earth—that is, through what we perceive in the form of earthquakes or geysers—acquire great importance, in that such phenomena have in various ways influenced the life of man. In the same way the study of geology is very closely connected with that of geography. The geognostic quali-

fication of a given country bears most heavily on one of man's most necessary occupations, on agriculture, and it is evidently of great import that substrata of quartz, *e.g.*, convey to some soils a greater amount of heat and looseness, thereby adapting them in a higher degree for agricultural purposes, or that the colour of certain layers is of great influence on the vegetation. Through geology, then, the geographer is led to a serious study of mineralogy and, partly, of chemistry; just as his regard for the astronomical position of the earth naturally induces him to a somewhat elaborate study of pure and applied mathematics. And since man's natural history can only be studied by an investigation of his past, the geographer cannot but embrace history proper as part and parcel of his vocation. By history, however, so vast an expanse of phenomena is meant, that we have already, at this preliminary stage of our attempt at a delimitation of geography, apparently entailed upon the geographer a task superior to the efforts even of the most gifted and most persevering. Should we now add, as add we must, that a due consideration of the vegetable and animal worlds of the earth cannot be avoided by the student of geography—that therefore he must be versed in the vast sciences of botany and zoology too—the question not unnaturally arises: What is geography not? And can it be anything definite, since it seems to be, or affects to be, everything?

In that predicament we are succoured by a simple observation. It is quite true that the geographer is obliged, in due pursuance of his researches, to diverge into a great number of complicated sciences. In doing so, however, he need not attempt to weld into one the huge masses of divergent knowledge. His is not the encyclopædic object of the philosopher proper. He does not try to elicit, from underneath innumerable facts and events, the basal principle; he only wants to determine the locus of all the events and phenomena caused by the interaction of man and earth. Geography is *per eminentiam* the science of localisation.

Metaphysicians have, as is well known, long fallen foul of space, and reduced it to a mere form of thinking. To common experience, however, the dignity, prestige, and power of localities, if not of abstract space, can

be no subject of any doubt whatever. With a slight alteration of the famous words of the Roman poet, we may say, *est locus in rebus*; human events are largely caused by that powerful geometry and mechanics of situation which can be shown to have worked the major portion of history. Some countries, like Dalmatia, Istria, Corsica, Ireland, Silesia, Crete, and others, have at no period of history contrived to play a decisive part for any considerable length of time, although many of their individual sons rose so great, and in the case of the Emperor Diocletian, a Dalmatian, and Napoleon, a Corsican, to the greatest possible eminence. Other countries again have, without courting it at all, attained to a position of supreme power in the teeth of the mediocrity of their sons, of intestine wars, or of errors that had repeatedly proved fatal to other nations. Who can, in this broad and secular fact, fail to recognise the power of the locus, or the geographical element in history? Like the ancient geometer, nations desirous of historic influence might ask of Providence, Give me a place where I may settle? Given the geographically significant locality, any white nation might have done what certain happily-situated nations now complacently ascribe to their own superior genius. The history of situation is, indeed, immeasurably more important than the history of races. The work of the strategy of situation has done some of the greatest deeds of history, and it may, in opposition to the usual overestimation of the element of time, be insisted upon that the element of space and locality is to history what the bass is in music, the dominating and colouring factor. Whenever we study a problem of history, past or actual, we can scarcely do better than array our facts, first of all, geographically. The incredible amount of work spent on the history of feudalism, for instance, might have been at once reduced and rendered more efficient by drawing up accurate and complete maps of France, Germany, etc., showing the localities where the *franc allen* or non-feudal systems of real estates were in existence during the eleventh, twelfth, and thirteenth centuries. By neglecting the construction of such maps we have, as is at present but too painfully evident, deprived ourselves of the means of thoroughly understanding feudalism. It is likewise with very many, if not with all,

other problems of history, taking this term in the widest sense of the word. How much might be gained by accurate maps showing the distribution of various systems of law and legal usages? There are some such maps for Germany, but they are all too general and vague; for England or Scotland they have not yet been attempted at all. In political economy more care has been bestowed upon the geographical element, and works like Meitzen's *Soil of the Prussian Kingdom* have shed a flood of new or clearer light on many a question of economics. The nature and history of diseases is also a subject by the geographical treatment of which our resources of investigation are considerably increased; and the geographical distribution of languages and dialects has, from its bearing on the origins of nations, a decisive importance for the student of the science of language.

In short, there is no subject interesting the student of man as a social being that can afford to neglect the geographical element. In peace, geography is paramount in commerce, industry, colonisation, trade, and in nearly all social sciences; in war, an adequate knowledge of the geographical conditions and measures of roads, terrain difficulties, and climate has more than once saved empires, as ignorance of geography has been visited upon innumerable generals with ignominious defeats or barren victories. The essence of the great Napoleon's genius was geographical. The chart of Europe that he carried about in his head was superior in accuracy and detail to any hitherto printed single map of that continent, and his immense power of organisation was mainly a power of duly localising the persons and institutions of his empire. Nearly all his unparalleled victories were essentially strategical, that is, victories won by an intense focussing of geographical considerations. On the other hand, the immense increase in the foreign trade of modern Germany—this one of the new and powerful factors in international politics—is largely due to that systematic and comprehensive study of geography which for the last two generations has been a distinctive feature of German national education. We may regret, but certainly cannot doubt it, that German works form by far the major portion of all the works published on matters of

geography, and that in their text-books, monographs, periodicals, and maps the Germans possess the most valuable amount of accurate information regarding any part of the globe or any community of men.

Geography, then, is mainly the study of the earth as a continuous series of localities, as an association of loci, or places where earth and man meet for constant mutual action and reaction upon each other. Hence oceanography, or the study of the physical constitution of the seas, cannot claim as large a space in a general geography as will the physics of the continent; nor is it necessary to give exhaustive descriptions of portions of the earth where man has not yet come into close contact with Nature. Such features of the earth may fitly be left to geophysics proper. On the other hand, even a short work on general geography must take into due consideration the distribution of human settlements on the surface of continents and islands. This, one of the most interesting and instructive aspects of geographical study, has so far not been much advanced, and cannot therefore be treated with all the broadness it requires. We are still ignorant as to the causes determining the situation, for instance, of the capitals and large towns of Europe, and we are likewise unable to account for the direction of roads, or the forms, number, and localities of rural settlements, in many parts of Europe.

From the preceding remarks upon the object of geography, it will now not be difficult to determine the single branches of that science; and since the geographical study of the earth is directed mainly upon the cosmic position, the morphology of the earth, and the geographical distribution of plants, animals, and man, geography may with fairness be divided into four sections:—

(*a*) Mathematical or astronomical geography.
(*b*) Geophysics.
(*c*) Geographical distribution of plants and animals.
(*d*) Geographical distribution of man, or the geography of man's dwelling-places, and man's social activities.

(a) *Mathematical or Astronomical Geography*

In treating of this, the first and preliminary branch of geography, we are again confronted with that peculiar difficulty of all truly geographical studies, which was mentioned at the outset of this work. Astronomical or mathematical geography is a "mixed" study. It is based on astronomical facts and theories as well as on purely terrestrial data. Overlapping, as it does, two vast provinces of knowledge, it has, especially in England, met with the fate of all enterprises encroaching upon different pursuits simultaneously. To the present day there is no elaborate, up-to-date special treatise on that subject in England, and many people will, as experience has taught us, wonder what may be meant by the term "astronomical or mathematical geography." Yet no serious study of geography can be thought of without a clear conception of the object, the methods and theories of mathematical geography. While in all geographical researches, as has been remarked, our main object is localisation, in astronomical geography that main object becomes the sole end in view. It is a study of the mathematical means of accurately determining any given point on the globe, so as to fix its position unequivocally, thereby enabling any subsequent student to recover the precise place of that point. The earth being, for all practical purposes, an equally rounded body, with no particular point standing out as one from which to start measuring distances, it is evidently no easy matter to devise methods whereby a given point can be accurately located, and its position permanently fixed. In all measurement of distances, however, there must needs be some fixed point with regard to which two or more distances may be compared. If now such a permanent starting-point be not easily found on the earth itself, it is perhaps possible to find such a point in the heavens.

Before proceeding in our investigation, we must ask our readers to ignore completely what, in previous times, they may have learned, or think that they have, with regard to our earth. Unfortunately for a

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thorough and really clear understanding of mathematical geography, very few people will frankly acknowledge their ignorance of certain general facts in the cosmic life of the earth, a knowledge of which is commonly held to be part of a gentleman's education. That the earth is round, and that it rotates round its own axis and revolves round the sun—all this and a few other general facts of mathematical geography are amongst the things of which most people would be ashamed to confess ignorance. Yet, for serious purposes, nothing is more important than a frank confession of that ignorance in the first stages of the study of astronomical geography. As in so many other studies, it is highly advisable in our present study too, to follow the historical way in which that study has been developed in the course of the ages. At first, most of the ancients thought that the sun, together with all the other stars, was moving round the earth. This is what we are taught by our eyes. Far from being an advantage to browbeat the testimony of the clearest of our senses, it is, as at present all great teachers of geography have acknowledged, a positive hindrance, if people enter on the study of astronomical geography with an alleged knowledge of the deception of our senses, and of the true movements of the earth and the other stars. The reason why so few people ever attain to a well-grounded insight into the principal teachings of astronomy must be found in presumptuous ignorance, by which they are prevented from the very attempt at clearing up these teachings for themselves, not by means of words repeated parrot-fashion after their text-books, but by clearly perceiving first the lessons of the senses, then by seeing for what reasons these lessons are incomplete and deceptive, and finally by comprehending the objective truth of the principal teachings of astronomy. As James Watt said, what we need most is "a book of blots," meaning, a book in which all the erroneous or incomplete researches of the great thinkers in science are given, showing the stages from darkness to dawn and light through which those heroes of thought had to struggle before they reached the final solution of their problems. The inestimable value of Kepler's, the great astronomer's, work on the star Mars is owing as much, if not more, to his revealing in

it all the preliminary or erroneous stages of the immortal discovery of his "laws," as to the communication of these laws themselves.

In the following pages we shall therefore at first *not* presuppose any knowledge of the real movements of the stars, but only follow the simple and apparently certain teaching of our senses. Then we shall show why that teaching cannot be accepted as the true one, and finally, what the real movements of the stars are like.

It was said above that our sole object in astronomical geography consists in so determining by means of a mathematical formula the precise location of any given point on the globe, as to enable any subsequent student to fix the situation of that point on the earth itself with unfailing accuracy.

Were the earth a flat body of no considerable dimensions, it would be easy to determine the position of any point. In our ordinary life we are so accustomed to determine situations of points on level plains, that we should have only to repeat our method of fixing the position of a house in a town, or an object in a house, in order to determine the position of any given point on that flat earth. The position of a certain house, say the Bank of England, is determined by naming the streets at the crossing-point of which the bank is situated. In America, for instance, where the streets of all towns built in the present century are laid out at right angles, the situation of any given house can be exactly determined by naming the two streets at the corner or crossing-point of which the house is situated, or by giving the number of houses, counting from that corner-house to the house in question. As in this case our measurements refer to straight lines only, so in nearly all cases of practical life. Whether we want to determine distances, heights, bulk, lengths, or widths, we invariably apply measures proceeding on straight lines, or bounded by such. Even in the case of dry measures, where circular lines are used, the ordinary man is no way enabled to test the measure, which he simply takes on trust. This inveterate habit of ours makes every attempt at bringing home to ourselves measures other than such as are in the form of or bounded by straight lines a very awkward labour. Having been used

to live, so to speak, in a world of straight lines, it becomes painfully puzzling to represent to ourselves a world in which measurement goes by curved lines. Yet this is what happens whenever we want to study the earth and the heavens. The space in which we perceive the stars evidently forms an immense globe, that is, a world as different from ours as is the circle from a square. Accordingly it is, as it were, unnatural for us to find our bearings in a world the dimensions of which so radically differ from those of our usual world. We can, with a little effort, clearly perceive some of the essential differences between a circle and a figure bounded by six or eight straight and equally long lines. But when, on leaving the plane, we try to conform ourselves to the relations of points, lines, and planes in a globe or sphere, we immediately find that some of our oldest mental habits are undergoing a strain which renders easy accommodation of our perception of space-relations very difficult.

Leaving therefore our original object out of view, we shall first try to acquire a ready perception of spacial relations in a globe by studying what the Greeks called sphærica, or the curious nature of the sphere or globe, and of the relations and proportions of points, lines, and planes on or in globes.

It may be taken for granted that the terms diameter, radius, segment, chord, and arc are familiar to the reader. Now one of the first remarks to make with a view of bringing out the spacial relations of a globe or sphere very clearly is to the effect that, although we can, in a globe no less than in a circle, draw straight lines from one point to another, yet such lines do not aid us at all in the perception of the spacial relations of a globe, the number of circles to each of which such a straight line may serve as diameter being boundless. On the other hand, the aid to perception which we require may be readily found in a plane intersecting the globe, each segment of that kind being a circle dividing the globe in two unequal, or, if it passes through the centre of the globe, in two equal parts. If, for contrast's sake, we pass such a plane through the longitudinal axis of an egg-shaped body, the segment so obtained will not be a circle (see Fig. 1, where *a* is the non-circular, *b* the circular segment of an egg-

shaped body, and compare that with Fig. 2, where segments are always circular, but varying in size according to their distances from the centre of the globe).

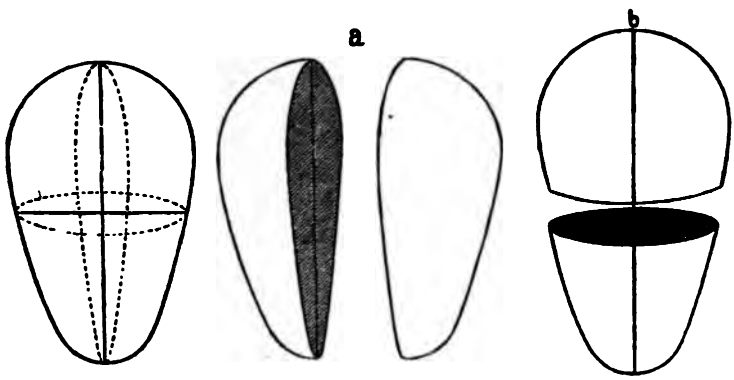

Fig. 1.

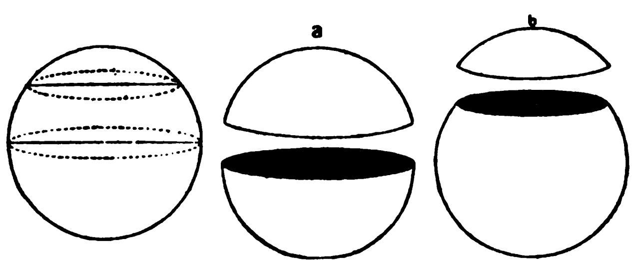

Fig. 2.

It may be seen at once that some circular planes (or circles) pass through the centre of a globe; others again do not. Such as do pass—and their number is unlimited—are called *great circles;* the others *lesser circles.* Two lesser circles may have to each other, or each to a great circle, the most varying positions; they may be parallel, or one outside the other, or one inclined against the other, or one intersecting the other. If two

lesser circles intersect, *one* of them may cut the other in two equal halves, but they cannot halve each other; nor can a great circle be halved by a lesser circle, although the reverse is the case. Two great circles, on the other hand, always intersect and halve each other, both having the same centre and the same diameter.

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ASTRONOMICAL OR MATHEMATICAL GEOGRAPHY

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ASTRONOMICAL OR MATHEMATICAL GEOGRAPHY

The Drawing of Parallels and Meridians on a Globe.—Any number of parallels and meridians may be drawn or imagined to be drawn upon the surface of a globe, but it is the usual custom to accept 180 parallels and 360 meridians to correspond with the division of the circle into 360 degrees. A meridian is drawn, and, this being a semicircle, is divided into 180 degrees. Through each of these points of division a parallel circle is drawn, each circle being perpendicular to the axis, making in all 180 parallel circles. The central one of these parallel circles is called the equator. One of these circles, the equator by preference, is then divided into 360 degrees, and through each of these points of division the meridians are drawn from pole to pole. These lines are easily drawn on a globe, especially when it is a globe mounted in a framework. The meridian on a model of the globe is represented by a vertical brazen graduated circle. A wooden ring going round the globe represents the rational horizon, and the globe turns on an axis, the ends of which are the poles. To draw the parallels on such a globe, we hold a pencil successively at each of the points of division marked on the meridian ring, and gently turning the globe, holding the pencil steady, the parallels are traced. To avoid a superabundance of lines, they are generally drawn at intervals of 10 degrees. When the parallel circles have been drawn, we proceed to divide the equator into degrees. With the assistance of the horizon ring above mentioned, the degrees marked thereon are easily transferred on to the equator; all we have to do is to place the globe so that the equator coincides with

the horizon ring. This done, the brazen meridian circle serves as ruler for drawing the meridians from pole to pole through the successive points of division on the equator.

To determine the Position of a Place on the Terrestrial Globe.—With the assistance of this network (Fig. 3), the position of any place on the surface

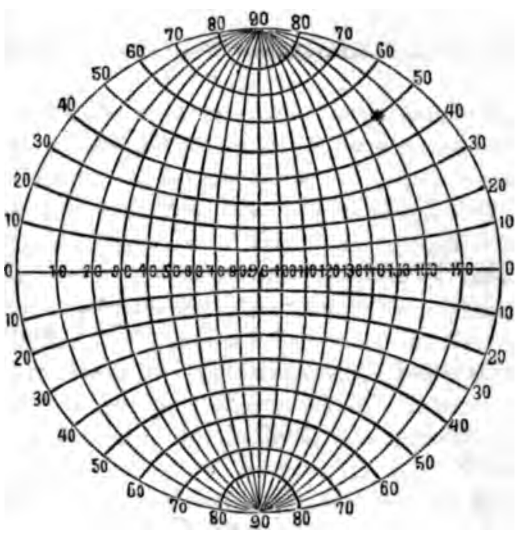

FIG. 3.—Hemisphere. Parallels and Meridians drawn at intervals of 10 degrees. The point marked has latitude 50° N, longitude 160° E.

of the globe can be accurately determined. It is usual to distinguish the poles of all globes as north and south poles. We count the parallels, beginning at the equator, towards the north and towards the south pole, from 0° at the equator to 90° at either pole, and the number of the parallel circle on which a place is situated is called its *Latitude* (from Lat. *latus*, broad), hence we distinguish north and south latitude according as a place is nearer the north or south pole respectively. Since all meridians are alike, any one may be selected as first or zero meridian,

and from this meridian, to the left or to the right, we count 360 meridians, so that the zero meridian is also the 360th. The point indicating the position of a place on the meridian is called its *longitude* (from *longus*, long), counted in degrees east or west of the prime or zero meridian,[1] from 0° up to 180°. Given the latitude and longitude, the required point must lie at the point of intersection of the respective parallel and meridian. For instance, if we say a point has 50° north latitude, 160° east longitude, we count from the equator, on any meridian, towards the north pole, 50°, and then along the equator, 160°, beginning at 0 meridian, towards the east. Where these two lines intersect one another the required point is situated. If we wish for greater accuracy, we divide the distance between any two successive parallels, and any two successive meridians, each measuring one degree, into 60 parallel circles and meridians, which are called minutes, and any of these again into 60 seconds. Astronomers carry this minute determination into tenths and hundredths of seconds. The position of points on the "celestial sphere" is determined by means of *declination* and *right ascension*. Declination of a heavenly body corresponds exactly to latitude of a point on the earth's surface, and is like latitude, north and south, whilst right ascension corresponds to longitude, but is always measured one way round from 0° to 360°, or more usually (since the apparent revolution of the heavens is completed in 24 hours) in hours, minutes, and seconds, one hour of time corresponding to 15° of arc.

A Revolving Globe and an Immovable Great Circle.—We imagine the eye placed in the centre of a hollow globe, from whence it could look round in all directions and survey the entire (inner) surface of the globe. In the case of the celestial globe, we can only survey the one (the visible) hemisphere, because of the existence of the horizon. It is clear that the different phenomena appearing during rotation depend in this case entirely upon the position of the axis with regard to the horizon.

To get a somewhat clearer idea of what the horizon is, we imagine an

[1] The meridian passing through Greenwich is universally taken as the zero meridian by English geographers, and also by many foreign nations. The French, however, prefer Paris.

immovable plane passing through the centre of our revolving globe, equipped with its network of lines. This plane we will call the horizon. It is not impossible, but rather difficult, to construct a hollow globe large enough to admit of our taking up our position in the centre of it on the disc of the horizon, but it is quite sufficient for all purposes of illustration to use a well-made globe, preferably a so-called induction globe.[1] We imagine ourselves stationed in the centre of the globe, but we see in reality the external, not the internal, surface of the globe. The horizon ring, which we imagine to form a complete disc cutting through the centre of the globe, divides the globe like the sensible horizon into a visible and an invisible hemisphere, and thus all relative positions become clear. We now place the globe in its framework, so that the axis coincides with the sensible horizon, and the poles consequently are on a line with the horizon ring; or we place the globe so that the axis is perpendicular to the horizon, in which case one of the poles rests in the hole at the foot of the frame; or lastly, we place the globe so that the axis occupies some intermediate position. The first position is called the *parallel sphere*; the second, the *right sphere*; and the third, the *oblique sphere*.

Horizontal Position of the Axis.—In this position (Figs. 4 and 5) the axis coincides with the meridian. The north pole is in the north, the south pole in the south point of the horizon. The equator passes through east point, zenith, west point, and nadir, and is perpendicular to the horizon, and all other parallel circles are also perpendicular to the horizon. Now considering there are 90° of the horizon between east and north point, and between east and south point, the first parallel circle must pass through the first degree marked on the horizon ring, counting from east point, the second through the second, etc. The ninetieth (the pole) therefore lies on 90° of horizon (north point and

[1] Induction globes are globes with specially prepared surface from which all pencil-marks can easily be effaced, as from a slate. For purposes of study only such globes should be used, with full complement of movable framework and rings. Fixed globes, on a framework without horizon ring, with the axis perpendicularly fixed or inclined at an angle of 66½°, are practically useless.

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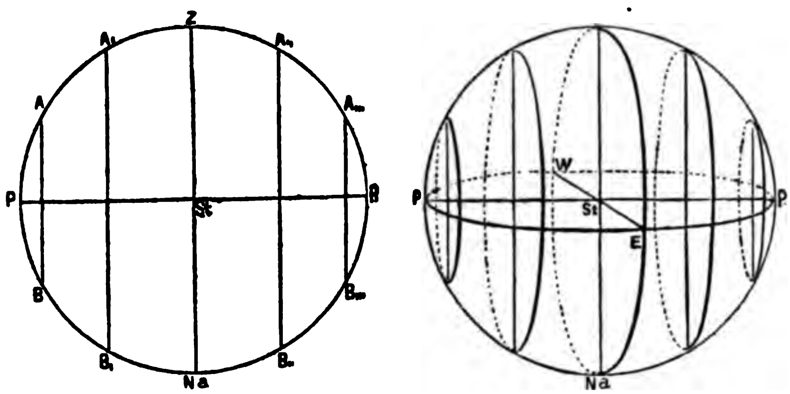
south point). The same thing occurs on the meridian circle when divided into degrees beginning at zenith. Not only the equator, which, like the horizon, is a great circle, but all parallel circles are cut by



FIGS. 4 and 5.—Right Sphere. *St*, position of observer; *PP*, axis; *EW*, east and west line; *Z*, zenith; *Na*, nadir; AB, $A_{,}B_{,}$, $A_{,,}B_{,,}$, $A_{,,,}B_{,,,}$ parallels of latitude.

the horizon into two semicircles. Let any one meridian coincide exactly with the eastern half of the horizon, the ninetieth meridian counted from there will coincide with the middle of the visible part of the meridian circle, the hundred and eightieth with the western half of the horizon, the two hundred and seventieth with the middle of the invisible part of the meridian circle. Turning the globe, we find :—

1. On the eastern half of the horizon all points rise perpendicularly.

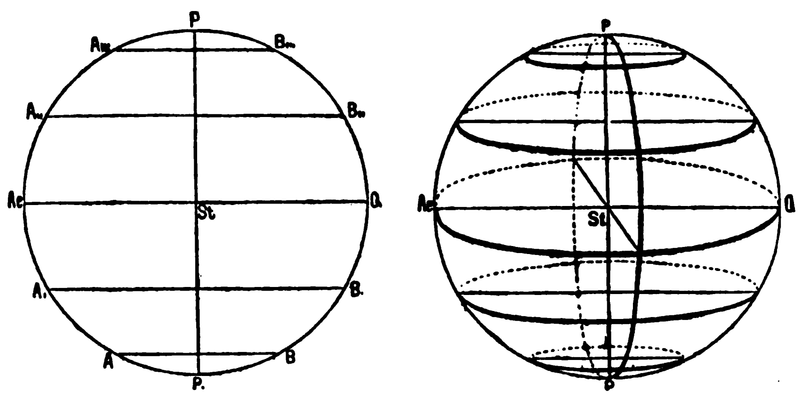
2. The place where a certain point rises lies exactly as many degrees from the east point to north or to south as the amount of degrees of its northern or southern declination.

3. When a fourth part of the revolution has been accomplished, all points which rose at the same time culminate on the meridian, *i.e.* reach their highest points when on that circle.

4. The distance from the zenith of any given point at the time of its culmination is equal to its latitude (northern or southern as the case

may be); hence the meridian or latitude (either northern or southern) must equal the complement of the zenith distance, or in other words, must be equal to the remainder when the zenith distance is subtracted from 90. Thus the meridian or latitude of a given point of 30° declination is equal to $90 - 30 = 60°$, northern or southern as the declination (or latitude) is northern or southern.

5. When another fourth part of the revolution is accomplished, all



FIGS. 6 and 7.—Parallel Sphere. *AeQ*, diameter of equator; *PP*, axis containing poles.

these points set again at the same time, and the west distance is again equal to the declination.

6. Every point remains exactly half the time of rotation above the horizon (visible), and half the time below the horizon (invisible), because the circle described during one revolution (diurnal circle) is divided by the horizon into two equal parts. Hence we say: all day and night arcs are equal, each containing 180°.

7. During one revolution all points of the globe become successively visible.

Vertical Position of the Axis.—The vertical position of the axis is just the opposite of the horizontal (Figs. 6 and 7). In this position the axis

coincides with the vertical, the poles with zenith and nadir, the equator with the horizon. The meridians are perpendicular to the horizon and coincide with the vertical circles of altitude; the parallel circles lie parallel to the horizon. Turning the globe on its axis, we find that:—

1. No point either rises or sets. There is no distinction of the cardinal points.

2. All points on the equator revolve on the horizon and never leave it.

3. Points in other parallel circles revolve at ever equal distance from the horizon; they do not culminate, and amplitude and declination are alike. Hence we say: the diurnal arc equals the diurnal circle; the night arc is 0.

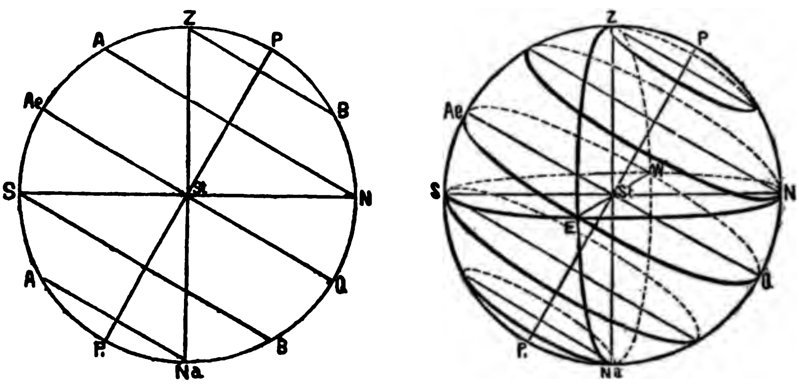
4. The nearer a point is to the horizon the greater its velocity, the nearer to the zenith the slower its motion, because the same point always remains at the zenith.

5. The points of the one hemisphere are continually visible, those of the other never.

Oblique Position of the Axis—Latitude.—While there is only one horizontal and one vertical position of the axis, there are an endless number of oblique positions, according as the axis slants more or less towards the horizon. We determine these different positions by stating the angle made by the axis with the meridian (angle of inclination of the axis towards the horizon), or what is the same, by giving the degrees of the meridian between the pole and the north or south point of the horizon. This distance is called *latitude.* The nearer it comes to 90°, the greater the latitude, and the more the phenomena resemble those of the vertical position of the axis. The nearer it comes to 0, the smaller the latitude, and the more it resembles the horizontal position.

In order better to understand the many difficulties arising from this oblique position of the axis, Fig. 8 gives a section of the globe on the meridian, coinciding with the meridian circle (this meridian is necessarily ever changing, because of the rotation). Let *PP* be the axis, situated in the centre of the meridian circle (hence the centre of the plane of projection). *ZNa* is the vertical, *SN* the meridian line, *St* the point of

observation. The straight line *AeQ* at right angles with *PP* represents the diameter of the equator on the meridian circle; and, in the same manner, *AN*, *ZB*, *ANa* are diameters of parallel circles on the meridian circle. The one half of the globe is above the plane of projection, with the east point in its centre perpendicular to the plane of projection, and a semi-diameter distant from *St*, the other half is below the plane of



Figs. 8 and 9.—Oblique Sphere. *S*, south; *N*, north; *E*, east; *W*, west; *SN*, meridian; ∠ *PStN* or *PN*=latitude; *ZStP* or *ZP*=zenith distance of pole.

projection, with the west point in the centre. Hence the angle *PStN* or the arc *NP* is the latitude, in this case 60°. We have thus:—

∠ *PStZ* or arc *PZ*, zenith distance of the pole.
∠ *ZStAe* or arc *ZAe*, zenith distance of the equator.
∠ *AeStS* or arc *AeS*, altitude of the equator.

For the sake of brevity we indicate latitude by p (here 60°), distance between the pole and zenith by q, distance from the equator to zenith by b, altitude of the equator by a, as illustrated in the following formula:—

$p + q = 90°$, because the vertex line is perpendicular to the meridian.
$a + b = 90°$, for the same reason.
$q + b = 90°$, because the axis is perpendicular to the equator, and hence to its diameter.

Therefore $q = 90° - p$; in our illustration $q = 90° - 60° = 30°$.
$b = 90° - q$; here $b = 90° - 30° = 60°$.
$a = 90° - b$; here $a = 90° - 60° = 30°$.

Or, in other words, latitude and zenith distance of the pole together make 90°. Therefore, if one of the two is known, the other is found by subtracting it from 90°. The greater the latitude the smaller the zenith distance from the pole, or the nearer to the pole the nearer to zenith. Zenith distance from the pole and the equator also make up an angle of 90°. The nearer the pole is to the zenith, the farther from the zenith is the point where the equator intersects the meridian circle. In like manner, also, zenith distance of the equator and altitude of the equator make up a right angle. It is clear, however, that latitude and zenith distance at the equator must always be the same, viz. $p = b$, because both make up an angle of 90° by adding the zenith distance of the pole; and, in the same way, zenith distance of the pole and altitude of the equator must always be the same, $a = q$, because both make up an angle of 90° by adding the zenith distance of the equator. Therefore the lower the pole, the higher the culmination point on the equator (point of intersection with meridian), and *vice versa.*

Different Length of Day and Night.—We will suppose the globe to be revolving in latitude 60°. On the east side of the horizon all points will be seen to rise at an angle of 30° obliquely to our right; they rise until they reach the culmination point (meridian) and then descend until they set on the west side of the horizon. All points on the equator rise in the east point, culminate when a fourth part of the revolution is accomplished at an altitude of 30°, and set in the west point after completing another quarter of their revolution. Their day and night arcs are of equal length, each completing half a circle (180°). Thus stars on the equator rise in the east, reach a greatest altitude of 30°, and are seen due south, set again in the west, the time from rising to setting being 12 hours.

Of points with southern declination, less than half of their daily path lies above, more than half below, the horizon. They are visible above the horizon for less than half the time of their revolution, and invisible below

[illegible] of their revolution. Their [illegible] of the equator by the amount of [illegible] with 10° south declination culminate at [illegible] 20° south declination culminate at 10°, and [illegible] at 30 – 30 = 0°—*i.e.* when they culminate, [illegible] altitude, they only just touch the horizon, [illegible] a moment. With still greater south declination [illegible]

[illegible] declination have longer day arcs than night arcs, [illegible] much longer as their declination is greater. They [illegible] as much above 30° as is the amount of their [illegible] with a declination of 10°, they culminate at an altitude [illegible] with 30°, 60°, etc. Since the 30th parallel circle is [illegible] the pole, and the pole stands 60° beyond the north point, [illegible] the 30th parallel circle must touch the north point, and [illegible] above the horizon; its day arc comprehends the whole circuit, [illegible] 0°. From its symmetrical position between east and [illegible] the same holds good for the east and west amplitude[1] of [illegible] north declination towards the north, as for those with south [illegible] towards the south.

In the same manner as we read off east amplitudes we can also read [illegible] proportion between day and night arcs of the different parallels. [illegible] how many degrees of a certain parallel are above and how [illegible] below the horizon. The points of intersection of the meridians [illegible] the parallels into degrees. Thus with 60° latitude we find—

Southern Declination.	Day Arc.	Visibility. (Parts of time of Revolution.)	Night Arc.	Invisible. (Parts of time of Revolution.)
5°	162¾°	0·45	197¼°	0·55
10°	144½°	0·40	215½°	0·60
15°	125°	0·35	235°	0·65
20°	102°	0·28	258°	0·72
25°	72°	0·20	288°	0·80
30°	0°	0·00	360°	1·00

[1] Amplitude is the complement of the azimuth or amplitude + azimuth = 90°.

With north declination the amplitudes of day and night arcs are reversed. Thus if the time of revolution be 24 hours, a point at 60° north latitude will be visible on the 20th south parallel circle for about $6\frac{3}{4}$ hours and invisible for about $17\frac{1}{4}$ hours, and in the 20th north parallel visible for about $17\frac{1}{4}$ hours and invisible for about $6\frac{3}{4}$ hours.

Circumpolar Points, Circumpolar Stars.—As stated above, the 30th northern parallel circle at 60° latitude touches the horizon with its lowest points in the north point; it lies entirely above the horizon. Its points never set, are always visible. Parallel circles at still greater distance from the equator, and therefore still nearer to the pole, also lie entirely above the horizon, but do not touch it, their lowest points lying beyond the north point on the meridian. The lowest point of the 40th parallel circle lies 10°, that of the 50th, 20°, etc., beyond north point. Hence, counting from the 30th parallel circle (at a latitude of 60°), all points describe ever-decreasing circles above the horizon. The centre of these circles is the pole. Such non-setting points are called circumpolar points (Lat. *circum*, round about; compare in the sky, circumpolar stars). These points move, like points that rise and set, from east to west, parallel between the pole and the south point, and then from west to east, below the pole star and between it and the north point. They are therefore twice visible on the meridian, once at their highest altitude, like all other points in their westward movement, and again at their lowest position, when journeying back to the east point. As already mentioned, culmination is the reaching of the highest altitude above the horizon, and this takes place when the point crosses the meridian. We might, however, more correctly put it this way: a point culminates when it crosses the meridian, and hence we say that circumpolar points culminate twice, making a distinction between an upper and a lower culmination. Every point culminates twice, but with points that rise and set the lower culmination is invisible. Even as, counting from 30th north parallel circle, all more northerly points are visible circumpolar points, so also, counting from 30th south parallel circle, all such points are invisible circumpolar points, because both culmination points are below the horizon.

The parallel circle which passes through the north point, in this case the 30th, separating the rising and setting points from the circumpolar points, is the boundary of the circumpolar points.

Phenomena to be observed while the Axis changes from the Horizontal into the Vertical Position.—In order clearly to understand the different phenomena to be observed while the axis is in an oblique position, we place our globe first with its axis in the horizontal position, and gently turn the globe from east to west, raising the one, say the north pole, while the other sinks, until the axis has attained the vertical position. We observe, in proportion as the one pole rises and the other sinks:—

1. That parts of the north parallel circles which were below the horizon rise above it on the north side, and equal parts of the south parallel circles sink below the horizon; so that of two symmetrical parallel circles on either side of the equator, the one gains in the length of its day arc what the other loses, or, in other words, that the day arc of the one equals the night arc of the other.

2. That the rising and setting points of northern and southern symmetrical parallel circles move at the same ratio towards the north or towards the south.

3. That the number of circumpolar points increases, the visible as well as the invisible. The boundary circles of the two kinds of circumpolar points lie at equal distances on either side of the equator.

4. That, up to a latitude of 45°, both the culmination points of the visible circumpolar points lie on the same side, between the zenith and the horizon. At 45° lat. the upper culmination point of the boundary circle lies in the zenith, the lower one in the north point. Above 45° lat. the boundary circle extends beyond the zenith to the other side of the globe; some of the circumpolar points therefore have their culmination points on opposite sides of the zenith. The boundary circle extends twice as many degrees beyond the zenith as the latitude exceeds 45°; thus for 46°, 2°, for 50°, 10°.

5. That the equator is the only parallel circle which always remains half above and half below the horizon.

It is, moreover, to be observed that in every position of the axis there are two modifications, according to whether the globe is turned in the one or in the other direction. But it is not necessary to distinguish these, as both appear simultaneously, the one above, the other below, the horizon. If we turn the globe to the right (the axis being in the vertical position), and imagine ourselves to be standing underneath the horizon with our head downward, so that our former nadir becomes our zenith, we get the vertical position of the axis with the motion to the left, and *vice versa*.

Other Phenomena which point to the Globular Form of the Earth. So-called Proofs that the Earth is a Globe

So-called Proofs that the Earth is a Globe.—Geographical school-books do not deduce the fact that the earth is a globular body, as we have done, from a comparison of phenomena at different points on its surface; but they simply mention certain phenomena as proof thereof. These phenomena are certainly closely connected with the globular shape of the earth, and with a complete knowledge of all facts bearing upon the subject, each one of them is conclusive evidence. But without this knowledge—which in most instances cannot be had until after the fact that the earth is a globular body has been established—they are valueless, and can at most only suggest the idea that the earth may be of a globular form. We will examine them more closely.

1. *The Horizon is always a Circle.*—It is stated that "all sections of th surface of a globular body must be circular, and if the globular body i of a great size, all sections of it will bear the appearance of a flat dis Therefore if the horizon, from any given point on the earth's surface, has the appearance of a flat disc, the earth must be round. Now we know that the horizon always forms a circle, and the circle of vision a plane, in the centre of which we stand; therefore the earth is a globe." However correct this conclusion may appear, it proves nothing in itself. Supposing all circles of vision together formed one gigantic plane, what would be the shape of the portion which we observed? It would be circular, of course.

We look round us in all directions as far as our eyesight reaches, at equal distance all round. This fact is not altered by the view being intercepted by objects rising above our circle of vision either perpendicularly or obliquely. For it is only over a certain and very limited space that we can judge of the distance of objects away from us; whatever is beyond our line of vision we instinctively project on to that portion of the celestial sphere which is behind our circle of vision, as on a plane of projection; and therefore even a square plane, as long as it is sufficiently large, would appear circular to us. We cannot even form a correct idea of the greater or lesser distance of near objects which stand out distinctly against the celestial sphere within our view; and contemplating a city at two or three miles' distance, we should be unable to decide which of its spires is the nearer and which the farther away from us.

Important Action of the Refraction of Light at the Horizon.—Another very important circumstance should be noted. The refraction of light in the atmosphere—of which more hereafter—has a very great influence upon the apparent shape of objects whose rays fall almost parallel to the circle of vision.

We shall see later on that the sun is visible to us before it rises above the horizon, and is still visible after it has set. Every one must have noticed and wondered at the seemingly flattened shape of the full moon at rising and setting. The Fata Morgana and mirage often reveal objects to us which are generally distorted and situated on the other side of the horizon.

In the face of these difficulties, the uniformly circular horizon line—which, moreover, can only be properly seen at sea or on a wide plain without elevated obstructions—cannot serve as a proof that the earth is a globular body. The uniformly circular form of the circle of vision is certainly a result of this fact; but in order to prove it, we must first deduct anything that really belongs to any of the above-named facts.

2. *The Circle of Vision increases in Size as our Point of Observation becomes more elevated.*—A somewhat stronger argument is furnished by the phenomenon that from an elevated position we observe a larger portion of

the earth's surface than when we occupy a lower position; but apart from the fact that this is not easily accomplished (as it means ascending in a balloon over a level plain, or failing that, climbing up to the top of an isolated mountain in the midst of a plain), the other matters mentioned in the last paragraph apply in exactly the same degree. And, moreover, one might suppose that from an elevated position the horizon would appear raised, but instead of that it appears hollowed "like a huge wash-bowl," and aeronauts who have reached considerable heights tell us unanimously that it appeared like being inside a hollow globe, consisting of the sky above and the horizon below them.

The same thing applies to the manner in which objects disappear behind us and rise up before us when walking. They disappear as they might be expected to disappear from a globular surface, the lower portions first; and they come into sight, the upper portions first; but this might perhaps be attributed to their upper portions being in a clearer atmosphere. The approaching and disappearing of vessels as we stand on the sea-shore shows exactly the same phenomenon.

The Shadow of the Earth at an Eclipse of the Moon.—The shadow of the earth at an eclipse of the moon,—no matter whether it be at rising, on the meridian, or at setting, before or after midnight,—is always round. "Now as only a round body, no matter what its position with regard to a luminous body, can throw a round shadow upon a projection-plane placed perpendicular to the axis of the conical shadow, this proves that the earth must be a globe." This statement would certainly be conclusive, if the moon were such a projection-plane. But the moon is a globe, as we know. Later on, when we have learned the distances of sun and moon, we shall be able to find by means of a simple calculation that the diameter of the earth's shadow cone at the distance of the moon is about three times the size of the diameter of the moon. The shadow surface is therefore about nine times larger than the disc of the moon. The consequence is that we can at any time only see a small portion of the shadow on the moon, as shown in Fig. 10; under the most favourable circumstances only about one-tenth. But one can easily see for oneself, and

it is moreover proved by the science of projection (Fig. 11), that the straight edge of an angular body can throw a curved shadow on to a sphere. Viewed thus superficially, therefore, the eclipses of the moon prove nothing for the globular form of the earth.

Journey round the Earth.—A further argument for the globular form of the earth is found in the supposition that, travelling in the same direction without turning back, one returns again to the starting-point. But such a journey, which would necessitate one's remaining within the same great circle, has never been accomplished, and can never be accomplished, notwithstanding the aid of steamers and railways. On any such journey

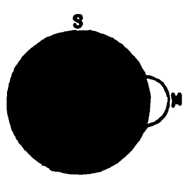


Fig. 10.—Section of the shadow cone of the Earth at the Moon's distance in true proportion. *S*, shadow; *M*, moon.

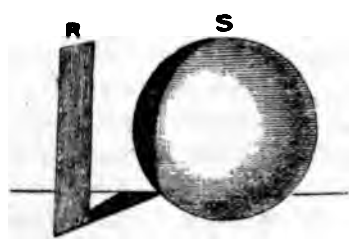


Fig. 11.—Shadow of Rectangular Surface thrown on a Globe.

one would meet with obstacles which would have to be evaded. They may be avoided by circumnavigation, or travelling round and returning within the same circle; but in order to be able to do this the globular form of the earth has to be presupposed. Moreover, the same phenomenon would also be present if the earth were of cylindrical form.

Conclusion by Analogy.—The conclusion by analogy, derived from the globular form of the other heavenly bodies, is no proof at all. Apart from the fact that we must first be convinced that all the other heavenly bodies are spheres, which from the earth's point of view can, at most, only be presumed for two of them—sun and moon,—it is not correct to say that all heavenly bodies are spheres, as is proved in the case of comets. From all this it is evident that only by comparing the different

phenomena at different stations upon the earth, as we have done, can one possibly arrive at any correct knowledge of the true shape of the earth. The so-called proofs give us only a dim apprehension, without any solid foundation for the building up of subsequent theories.

Objections to the Globular Shape of the Earth.—Some of the objections raised against the theory that the earth is a globular body have already been refuted in the explanations just given; others will be dealt with on a future occasion. To the former class we may reckon the objection that even at sea we see a straight horizon. Just as we can cut out of any circle so small an arc that it cannot be distinguished from a straight line, so we can cut a small portion from any globe, so as to be indistinguishable from a flat surface. Nevertheless we can under certain conditions bring the curvature within our line of vision—not on the sea-shore at the approach of a ship, in which case we do not know how much has to be put to the credit of the refraction of light, which is so strong at the horizon; but we can do it on a moderately large lake. Lying down on the one shore, and looking across, the objects on the other side lack a small, though quite noticeable, part of their lower portions.

To the other class belongs the argument, why objects on the other side of the earth do not fall off. Upon this point we will here only remark that experience teaches that this does not happen, but all bodies fall always in the direction of the vertical line. The grounds for this will be explained later on.

Conclusions drawn from Results so far obtained

Objections against the View hitherto held with regard to the Phenomena in the Universe.—So far we have supposed the earth to be a stationary body suspended in the midst of space, and the celestial globe, with everything in the universe external to the earth, to make one revolution round the earth in 24 hours, in the direction from east to west, while the moon and the sun (and a few other celestial bodies) described their own orbits, as a rule from west to east.

The thousands upon thousands of fixed stars did not alter their relative positions, and this helped to give the impression that the celestial globe was a solid body in which these luminaries were suspended.

There was nothing unnatural in this representation, so long as we knew nothing of the size and the distance of these celestial bodies. In order to form a good idea of the entire mechanism, we should have had to construct a large hollow globe with the fixed stars marked on it, an axis passing through the two poles, and in the centre a small globe to represent the earth. Outside the larger globe we should have had to fix two rings, in the positions occupied by the orbits of the sun and the moon in the heavens. One of these rings would have a lamp fixed in it to represent the sun, the other a dark globe to represent the moon. We should want three instruments to illustrate the different motions which we have so far considered. The one would have to turn the whole celestial globe (with the two rings fixed to it) round its axis in 24 hours; the second would have to move the moon round in the opposite direction within 27⅓ days; and the third would in a similar manner move the sun round its ring in 365¼ days.

We knew nothing of the causes of these motions and the forces at work, but the general plan was not contrary to the law of simplicity. Even the fact that, on account of the immense size of the celestial sphere, the points on and near the equator would have to travel with such enormous velocity does not destroy this simplicity, because this motion is the result of the revolution of the globe, and our mechanical contrivance makes the globe to revolve in that way. But this simplicity can exist only so long as all the constellations and sun and moon form part of one and the same sphere. Of the sun and the moon we know decidedly that this is not the case.

Continuation.—We will accept provisionally that the phenomena of the fixed stars may be represented as though they were attached to the surface of a globe, "the celestial sphere," and ask, How about the sun and the moon? They must in the first place describe a daily circle from east to west. When the moon is in the equator it would have to travel in one

day about one million five hundred thousand miles, or in one second about 17 miles. The sun in similar position, being 382 times farther away, would have to travel 382 times faster, *i.e.* about 6500 miles a second. At this rate the velocity of the moon in its daily orbit would be 80 times, that of the sun 32,000 times, greater than the velocity of sound in air. This enormous velocity of a body (in the case of sound it is a question of wave-motion) may well excite our astonishment; but so long as we know nothing of the causes and forces at work, there is no reason why we should not accept it as a fact. For although the velocity of the moon in this supposed motion is 80 times greater than that of sound in air, we shall learn presently that there are bodies which travel with still greater swiftness.

But both moon and sun have, besides their daily motion, yet another, opposed to it. Again, this in itself would give us no reason for doubt, if this second motion were exactly opposed to the daily motion; we should merely have to accept that the moon, instead of covering 360° per day, covers only 347°; and the sun, instead of 360°, only 359°—in other words, that their daily revolutions in the sphere fall relatively short by 13° and 1°.

But because of their backward or retrograde movement, both moon and sun are continually crossing other parallel circles. It is therefore clear that both these bodies, simply because of their varying position in the different parallels, must always be changing the rate of their velocity, for they are bound to accomplish their diurnal circle within 24 hours, and this circle becomes smaller as they move away from the equator, and greater as they get nearer to it.

This circumstance is of so great importance that we must endeavour to realise it as clearly as possible. The simplest way of illustrating it is the following. We trace on a globe the parallel circles and the ecliptic.[1] Supposing that on a certain day the moon is at the point of intersection of the ecliptic and the equator, it would on that day

[1] We know, of course, that the moon's and the sun's orbits describe two different great circles, but for our present purpose we need take no notice of that. The sun's apparent path is called the ecliptic, and the moon's path is inclined to this at a small angle of about 5°, so that we may regard these as the same for the purpose of this explanation.

have to accomplish a circle of 324,200 geographical miles, or in one minute 225·1 miles. Going back 13° 10′ to the east on the ecliptic, we find the point in the sky where the moon will be the next day, and we can read off from our globe (or calculate) that it will then be 5° 13′ north (or south) of the equator. But this is on a smaller parallel, and the moon will have to travel at the rate of 224·2 miles a minute, or 0·9 mile slower than the previous day. Continuing to follow the path of the moon, we register the following figures tabulated here:—

	Point in the Orbit (Longitude).	Declination (Distance from Equator).	Distance accomplished in 1 Minute (Miles).	Increase or Decrease as compared with Preceding Day.
1 Day	0°	0°	225·1	...
2 Days	13° 10′	5° 12′	224·2	0·9
3 ,,	26° 20′	10° 10′	221·6	2·6
4 ,,	39° 30′	14° 40′	217·8	3·8
5 ,,	52° 40′	18° 27′	213·6	4·2
6 ,,	65° 50′	21° 17′	209·8	3·8
7 ,,	79° 0′	23° 0′	207·3	2·5
8 ,,	90° 0′	23° 27′	206·6	0·7

We see from the above table that the rate at which the moon travels in its daily circuit decreases from day to day (or, more correctly, decreases constantly and irregularly). This is due to its change of position in the heavens, and so far we have not been able to give a reason for it. During the next following eight days the moon's velocity increases in reverse order, and the same appearances present themselves on the other side of the equator.

Similar phenomena may be observed in the sun's path, although, its apparent motion being about thirteen times slower, the relative increase and decrease in the velocity is necessarily smaller. On the other hand, owing to the sun's considerably greater distance, its absolute motion must be a very great deal swifter than that of the moon. The following table gives the sun's motion, not from day to day, but from ten to ten days. The velocity is given in seconds:—

	Point in the Orbit (Longitude).	Declination (Distance from Equator).	Distance accomplished in 1 Second (Miles).	Decrease as compared with the Previous Figure.
1 Day	0°	0°	1454·5	...
10 Days	10°	3° 58′	1451·0	3·5
20 ,,	20°	7° 49′	1440·9	10·1
30 ,,	30°	11° 29′	1425·4	15·5
40 ,,	40°	14° 49′	1406·1	19·3
50 ,,	50°	17° 45′	1385·2	20·9
60 ,,	60°	20° 10′	1365·4	19·8
70 ,,	70°	21° 58′	1348·9	16·5
80 ,,	80°	23° 4′	1338·1	10·8
90 ,,	90°	23° 27′	1334·3	4·8

Now as the sun and moon, at each point of their orbit, accomplish their daily circuit with the speed which each special point demands in relation to the revolution of the heavens, we are compelled to accept one common law and one common cause for the daily rotation of the sphere of the fixed stars, and the daily motion of moon and sun (and of the four other stars which have not yet been mentioned). This common centre we have up to now imagined to be the celestial globe, the one vault carrying all celestial bodies, with the exception of the earth. But as we have now seen that the celestial globe is not so constituted, and that, even still supposing the fixed stars to be in some way fastened to it, the sun and the moon (and those four other stars) are certainly not, we begin to feel serious doubts as to the correctness of our imagined system, and we try to find some other ground or cause in explanation of the phenomena we observe.

Continuation.—Moreover, we can hardly help feeling sure that there must be great differences in the distance of the fixed stars from us, and that they are not, as we at first imagined, mere illuminated points, but luminaries in themselves. We become convinced also that they must be of considerable size, being visible to us at such immeasurable distances. We conclude, therefore, that they, like sun and moon, are freely suspended in the universe, and that the blue background we imagined to be a solid is merely a reflection of their light. We no longer believe in a

material support for the heavens, any more than we do for the rainbow. It does not require a great effort to convince ourselves that the intense blue of the sky is dependent on the conditions of our own atmosphere, dependent on the rarity of the air (the deep blue of the Italian sky), dependent on the altitude of our position (the almost black sky of aeronauts at great heights), varying even in its colouring with the difference in the density of the air in one and the same place. In short, the blueness of the celestial sphere is merely an effect of reflection of light (Tyndall's explanation) or rather scattering by dust particles. The absolute impossibility of explaining the daily motion of all celestial bodies otherwise than by one common cause, compelled the ancients—believing that the fixed stars all formed part of one and the same sphere, in which sphere sun and moon could find no place—to accept a number of concentric, perfectly transparent spheres, one for each of the celestial bodies which they could assume to possess separate motion.

In order to explain the different phenomena which, so far, have come under our notice, by this system or theory, we should have to imagine three such transparent spheres (globes), one with a semi-diameter as yet unknown, for the fixed stars, one with a semi-diameter of 93 million miles for the sun, and one of 240,000 miles for the moon—all of which would have to be connected by some invisible bond, and together describe a rotation round the axis of the world within 24 hours. This is so childish a notion that we dismiss it at once, without bringing forward any further arguments.

Meanwhile the doubt is gaining ground in our minds, whether things really are as they appear to us, and we become more and more convinced that we labour under some optical illusion.

Optical Illusions.—All our senses are open to delusions. Even the so-called most materialistic of our senses, the sense of touch, with which we believe to hold direct communication with the external world, is liable to be deceptive.[1] To illustrate what I mean, place a little ball on a

[1] It is perhaps more correct to say that our interpretation of sense impressions is liable to be faulty, rather than that the senses are deceptive.

horizontal plane, say a table. Cross the middle finger over the forefinger, and spin the ball round in the angle formed by the tips of middle and forefinger. Notwithstanding the certain knowledge that there is only one ball, it will always feel as if there were two, no matter how often the experiment be repeated. Leaving the fingers in their natural position, in which case we can also feel the ball with two fingers, we shall never make that mistake. The phenomenon can easily be explained. If we try the experiment with the right hand, when the fingers are in their natural position, we take hold of the ball with the right side of the fore and the left side of the middle finger. But these sides are next to each other, and from constantly recurring experience we know instinctively that a body placed between these two fingers can be touched by both. Now when we cross the fingers we touch the ball with the left side of the fore and the right side of the middle finger, and it gives the impression as if there were two balls, one on the left of the fore and another on the right of the middle finger.

We will not at present pursue this most interesting subject of optical illusions, which are quite compatible with perfectly sound organs of sense, and perfectly sound nervous systems, and have no connection with morbid fancies or ideas. We will only take notice of such optical illusions as immediately concern our present purpose, and which are caused by different individual motions.

When in a station two trains are standing on neighbouring lines, and we are seated in one of these trains, supposing our train begins to move, without our being aware of the movement, *i.e.* without a warning jerk, it will seem to us that we are still standing still, and that the other train is moving on; and it is only when watching the wheels of the other train, and seeing that they are not moving, that we begin to realise that it must be our train that is moving. Supposing we are seated upon a horizontal plane, which moves evenly without obstruction upon a vertical axis, it appears as if all objects outside this plane turn round us in the opposite direction to that in which we are going. Similar experiments may be made in many different ways. Even when we are seated at a

window, and turn our head to and fro, it looks as if the window and all external objects are moving in the opposite direction.

The Phenomena of the Diurnal Motion can be explained equally satisfactorily by the Rotation of the Heavens as by that of the Earth.—The phenomena of daily motion can be explained equally satisfactorily by a rotation of the heavens round the axis of the universe from east to west, as by a rotation of the earth round its axis from west to east. Each one in itself is equally plausible, so much so, that if in some way or other the fact were established that there really is a solid celestial globe in which the fixed stars are suspended as luminous points, and sun and moon are merely discs, and some one nevertheless maintained that it is the earth and not the celestial globe which rotates, we should not even then be able to confute the one or to prove the other statement. All the ordinary phenomena appearing in the heavens in the course of one day are so fully explainable by either of the above theories, that our unaided sense of sight is wholly inadequate to decide between them. Other considerations must be brought to bear upon the matter, in order to come to a conclusion.

Improbability of the Rotation of the Heavens.—If we accept that the heavens rotate round an axis, we must premise it by stating that in that case at least 6000 stars (for that is about the number that can be counted with the naked eye), each differing in distance from one another and from us, scattered all over the universe, and not seemingly connected by any common tie, nevertheless move so unanimously that they all describe a certain circle within exactly the same period of time—not round the centre of the earth, but round an axis which is nothing but an imaginary mathematical line. If this were so, all the stars would have a different rate of motion; there would be velocities of millions of miles in a second, down to zero (at the pole), and their velocity would depend upon their distance from the axis (not of the earth), and secondly upon their position in the parallel circles. What a complication! If a mechanic were to construct a model to illustrate this, he would have to make at least 6000 different instruments, which would have to agree to a

hair's-breadth in this one point, that no matter how much difference there might be in the velocity of the different bodies, they must all accomplish one revolution in exactly 24 hours. These velocities, however, would at least be consistent; but what is the mechanic to do with regard to the sun and the moon? Their retrograde movements are performed round the centre of the earth; we should have to imagine for these bodies some sort of force centred in the earth, but how could instruments be made to give an accurate idea of the daily motions of sun and moon, which are for ever varying in their velocity? In truth, the supposition of a daily rotation of all bodies outside the earth is so contrary to all common sense that it is impossible for us to accept it.

The Rotation of the Earth causes the Daily Phenomena.—The whole question becomes perfectly simple, when we accept a rotation of the earth from west to east. All the diurnal circles of bodies outside the earth become then merely the result of the earth's rotation. What would be impossible to illustrate by any number of the most complicated instruments, can now be made perfectly clear with one single piece of machinery. We should require, however, two other instruments to show the paths of the moon and of the sun. There can be no longer any doubt about it that the apparent daily revolution of the heavens is caused by the rotation of the earth.

Objections to the Rotation of the Earth.—But is it possible that such a motion can exist without our being aware of it? For if this be so, every point at the equator would have to move at the rate of 1500 feet per second, *i.e.* would travel faster than sound; and we should be perfectly unconscious of such velocity. Are we to believe that when a sound is raised at the equator we should not be able to hear it, because we ourselves are carried through the air so much more rapidly? and supposing we were to take a leap in the air, even if it took only a quarter of a second, should the place under our feet in that period of time have to travel nearly 400 feet to east ahead of us? In London, whose latitude is about $51\frac{1}{2}°$, we should be whirled along at the rate of over 1000 feet per second—faster than the most furious hurricane; and we should be supposed not to become aware of this, if it were only by the cutting winds!

In St. Petersburg or Stockholm (latitude 60°) we should still maintain a velocity of 750 feet per second. And lastly, are we supposed to be also unconscious of the fact that, after travelling through the universe for 12 hours, we should reach the point which is now occupied by our antipodes?

At first sight these objections may carry some weight. But the motion of the earth round its axis, as we see from the apparent motion of the stars, is so regular, so even, that we know of no motion within our circle of vision which can be compared with it. No steamer, no railway train, can come up to it; and yet we do not feel the motion, even when the train goes at its fullest speed,[1] and in a comparatively clumsily moving compartment, and if we were carried asleep into a railway carriage, we should not be conscious of any motion at all when we awoke. We can therefore not experience it, neither can we see it, because everything that belongs to the earth, air, clouds, we ourselves, etc., all share it alike. We can only see the motion when watching the movements of those bodies which do not belong to the earth, the celestial bodies. Nevertheless, the phenomena of the trade winds are only explicable on the hypothesis of the earth's rotation.

The Power of Persistence in Bodies.—It will be advisable to inquire somewhat closer into this matter. When we see a body in a state of rest, and it begins to move, we immediately form the conclusion that some external force is the cause of this change; for we know from our earliest experience that no body can, of its own accord, alter its state of rest into motion (Newton, Law I.). And mature consideration shows us likewise that they cannot pass from a state of motion into one of rest, but rather that they can in no wise alter their state of motion, but must continue in the given direction and at the given ratio. This latter statement seems to be contradicted by experience; for we see that, without exception, every body set in motion upon the earth gradually slackens its speed until it finally stops. But we also see that in every individual case the slackening of speed is attributable to external circumstances and causes (obstruc-

[1] It is only the jerks or shocks that we feel.

tions), and by lessening these obstacles we can diminish the resistance; to remove the obstacles altogether is absolutely impossible.

If we compare the effect produced upon a cart when set in motion upon a rough road, upon a well-paved road, and upon iron rails, we see that if we could put aside friction, the resistance of the air, etc., the cart would go on without ever stopping. We believe that it is the nature of bodies that nothing in them can alter their condition of rest or of motion, and that this change only takes place when some external force is brought to bear upon them; this is called their inertia. This is fitly illustrated by the collision of two railway trains, when, notwithstanding the shutting off of steam and the application of the brake, accidents happen.

Motion of Bodies which are mutually connected.—When bodies are connected, and one of them is put in motion, the motion gradually spreads to all the others. When the bodies are all in motion, and one of them is brought to a state of rest, all the others must follow. When we are seated in a railway carriage, and the train is suddenly put in motion, we fall backwards; if the train is in motion, and is suddenly brought to a standstill, we fall forwards. In the first instance we remained at rest for an instant while the train went on; in the second we went on moving while the train stood still; both were the result of the fact that, because of the suddenness of the change, the time was too short to impart the condition of rest or of motion to our bodies. When the carriage passes gradually from its state of rest into that of motion, or the reverse, the new condition is imperceptibly conveyed to us.

All objects, therefore, connected with the earth share alike in its rotations—the air which carries sound, sound itself, we ourselves, and so on. The body which falls, we, when we jump, cannot tarry behind to westward, because the falling body and we ourselves have already in us a motion to eastward.

The same applies to a balloon thrown up vertically from a ship or a railway carriage; it does not fall down behind us, although both ship and carriage have moved on during the time which elapsed between the rising and the falling of the body.

Imperceptible Balance of Motion when travelling to North or to South.—Another objection against the rotation of the earth might be raised, namely, that although we may not be conscious of it as long as we remain on the same parallel, we must feel some change when moving from the place of our habitation to the north or to the south. In the first case (in the northern hemisphere) we touch points which move more slowly, and we therefore have a swifter motion than our surroundings; in the other case we bring a slower motion to bear upon the existing quicker

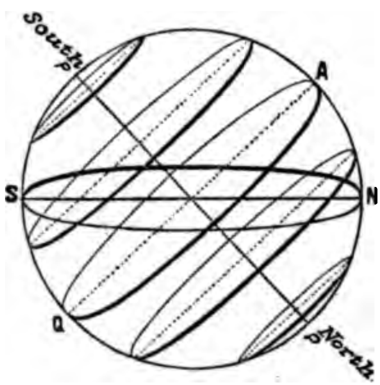


FIG. 12.—Horizon of Latitude 48° S.

motion. If bodies could move sufficiently rapidly from north to south, and the reverse, without passing through all the intervening points, this objection would be justified, but it is more like being in a railway train, with gradually increasing or decreasing velocity, but no sudden starts.

Appearances on the Earth which are a Result of its Rotation. (*a*) *Deflection of Cannon-Balls.*—Although from all that has been said it can be no longer doubtful that the earth rotates round its axis in 24 hours from west to east, it would certainly be very interesting if we could find some terrestrial phenomena in which we could recognise a proof of the

rotation of the earth. Such phenomena do exist, although (with the exception of one) they are somewhat difficult of observation. The instance given in the last paragraph is a phenomenon of this kind. When bodies move sufficiently rapidly from north to south, or the reverse, one may notice the effect of the rotation of the earth in them. Suppose a cannon-ball is shot in a straight line due north. If we stand facing the north, the terrestrial sphere turns from left to right. In the northern hemisphere the cannon-ball passes through parallels with slower

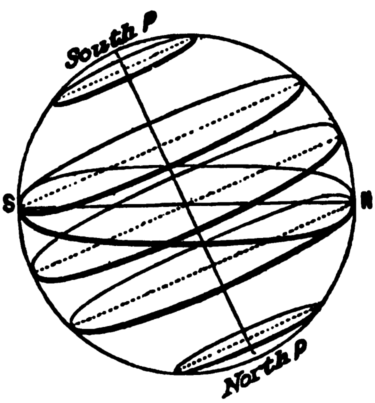


FIG. 13.—Horizon with 67½° S. Latitude.

motion. In the time which the cannon-ball needs to reach the disc, the cannon itself will move a greater distance to the right than the disc; but the cannon-ball has the velocity of the cannon, and will therefore strike the mark a little to the right. If the cannon be fired towards the south, we face the south, and the earth then turns from right to left. Now the cannon-ball passes through parallel circles with greater velocity. The disc therefore will get in advance of the ball, and the mark will be hit a little to the right. However small this difference in motion may be, gunners have noticed the fact, without being able to account for it, and indeed one can hardly guarantee the

absolutely perfect make of the barrel, to ensure of the shot being fired in a perfectly straight line.

(*b*) *Trade Winds.*—The trade winds illustrate the same thing, only in an intensified degree. The heated state of the air on and near the equator produces there a region of low pressure (rarefied air), into which the colder, denser air of the poles is driven. If the earth stood still we should have a north wind in the northern, and a south wind in the southern hemisphere. Supposing we stood in the direction of the wind, *i.e.* in the northern hemisphere facing south, the earth would move from right to left. Thus the air enters parallels which, moving with greater velocity, must therefore be deflected to the west (right); *i.e.* the north wind deviates to the west, and appears to blow from north-east; it becomes a north-east wind, north-east trade wind. Of course in the southern hemisphere the winds coming from the south must also be deflected to the west and become south-east trade winds. All appearances confirm this view, although they may be influenced by different meteorological occurrences, such as the configuration of continents, etc.

(*c*) *Deflection of Falling Bodies—Benzenberg's Experiment.*—The most serious objection against the rotation of the earth on its axis is the one already alluded to, viz. that a stone falling from a tower is not deflected to the west, as the earth's rotation seems to demand. We know that, because of the power of inertia in the stone, the velocity which it possesses in virtue of the earth's motion towards the east must be continued while falling. On inquiring more closely into this matter, we find that this falling of a body freely towards the earth will give us, as the great Newton first taught, a direct proof of the rotation of the earth, since the stone, contrary to the common belief, falls forward to the east. On account of the rotation of the earth, every point on it—the foot of the tower as well as the top—describes a circle in 24 hours. But the semi-diameter of the circle, and therefore the circle itself, described by the top of the tower is larger than that of the foot. Consequently the top of the tower and everything at that elevation moves faster than the foot. If from *T*, the top of the tower (Fig. 14), we let fall a stone, it will preserve its greater velocity while falling, and

reach the bottom a little to the east of point *S*, which forms a straight line with the top of the tower. If it were possible to let fall a body from a height of about 1000 metres, about the height of the Lindkogel near Baden, an insignificant height, compared to the earth's diameter, the eastward deflection would be nearly half a metre. But such heights are not at our disposal. Nevertheless, the experiment has been made several times. Benzenberg was not the first who tried it, but the first who was successful. After him it is called the Benzenberg experiment. He tried it in 1802, on the Michaelis Tower at Hamburg, and in the coal-shaft at Schlehbuch. In Hamburg, with a drop of 235 Paris feet, the experiment showed an eastward deviation of 4 Paris lines; in the second case, with a drop of 260 Paris feet, a deviation of nearly 5 Paris lines. Both these results agree as closely with theory as can possibly be expected for such difficult experiments. The eastern deflection at Hamburg, according to this estimate, is only 0·1 line out of its reckoning. In the year 1832 Reich tried the same experiment at Freiburg in Saxony, in the Dreibrüder shaft, which is 158·5 metres deep, and the result was a deviation to east of 27·52 mm.; out of 106 trials Reich found an average deviation of 28·4 mm.

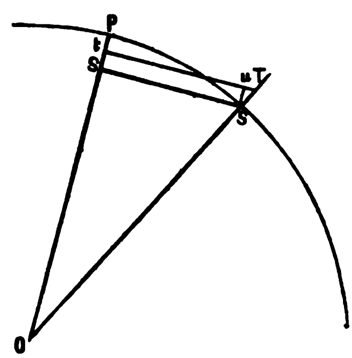


FIG. 14.

(*d*) *Apparent Alteration of the Plane of Oscillation of the Pendulum—Foucault's Experiment.*—As early as the year 1661 some natural philosophers in Florence discovered that a pendulum when put in motion slowly appears to alter its plane of oscillation. It was observed but not explained, until about the middle of the last century the French philosopher Léon Foucault succeeded in proving that the alteration in the plane of oscillation of the pendulum is merely apparent, and is in reality a consequence of the rotation of the earth. Owing to the power of inertia, a swinging

pendulum must preserve its plane of oscillation unchanged, even if the point of suspension should be slightly moved. A simple experiment which is easily performed will show this. Take a plate or table (Fig. 15), turning on a vertical axis, and raised on rollers to ensure a smooth and easy motion in all directions. Affix on the edge of the plate a vertical rod supporting at the top a branch rod swinging freely in all directions, on the under side of which little hooks are screwed at regular intervals, on which the pendulums are to hang. Hang a pendulum (consisting of a long thread with a leaden ball at the end) on to one of the hooks, exactly over the centre of the plate, put the pendulum in motion, and turn the plate round; it will be observed that, notwithstanding the motion, the pendulum does not alter its plane of oscillation. Turning the arm so that the point of suspension falls beyond the centre of the plate, it will be seen that the plane of oscillation of the pendulum moves parallel with it, and preserves the same direction. Even when the plate is moved out of its place in any given direction, the plane of oscillation always remains parallel to itself. Supposing I saw only the plate and the pendulum, or was myself seated on the plate, and I marked by a line the point where the plane of oscillation intersected the plate, the plate being gently turned round without my being aware of it, it would appear to me as if the pendulum had changed its plane of oscillation.

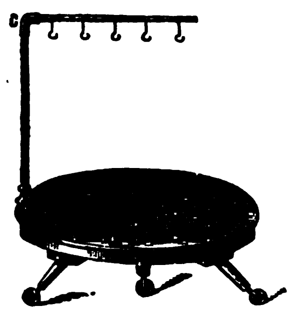


FIG. 15.

Suppose we are at the north pole of the earth, and there put a pendulum in motion, it will be found that the plane of oscillation makes in one day a complete circle from left to right, because in that time the earth completes one rotation from right to left. The deviation would be $\frac{1}{4}°$ per minute; at the south pole the motion would be in the reverse direction. At these two stations the point of suspension suffers no displacement, and the pendulum, when at rest, lies in the prolongation

of the axis of the earth. At all other stations on the earth the point of suspension performs daily a complete circle, increasing in size as it comes nearer to the equator. The plane of oscillation is moved from its place. In obedience to the law of inertia, it will try to preserve its original direction, but to do so it must constantly pass through the point of suspension and the centre of the earth.

At all stations on the equator, all lines facing the same cardinal point (for instance the north) are parallel to each other, because the meridians are parallel. A pendulum which is set swinging in the northern direction will always retain that direction, and the rotation of the earth will not be indicated by any apparent displacement of the plane of oscillation. All points between the equator and the pole towards the north (and the other cardinal points) gradually incline towards each other, according to the distance of the point from the pole. If the pendulum is set swinging in a given meridian, *i.e.* from north to south, its plane of oscillation will be deflected from the meridian, owing to the rotation of the earth, with a speed increasing in proportion as the place is nearer to the pole; at the pole itself it will be $\frac{1}{4}°$ per minute. This deviation can easily be calculated for every latitude, and experiments agree everywhere with these calculations. For the performance of such experiments large halls or churches are most suitable, but even in an ordinary room it may be tried with satisfactory results, the principles of the phenomenon being, at any rate, clearly discernible.

The simplest method is to take a long string, at one end of which a ball is fastened, and on the under side of which is a sharp point in direct prolongation of the string; when the pendulum, fixed with the other end to the ceiling, is at rest, this point or lead should touch the floor. We now draw a circle with the pendulum as centre, and at the edge of this circle we throw up a wall of finely-sifted ashes, so that when swinging, the sharp point, each time it reaches the wall, makes a little mark in it. In order to set the pendulum in motion, the ball is lifted up and a thin thread fastened sideways, by which it is held suspended; then the thin thread is burnt off, and the ball swings with regular oscillations through

the nadir of the pendulum. If the pendulum be set in motion by hand, it is difficult to avoid a side push or jerk, which would prevent the pendulum from swinging in a plane, but would make it describe a somewhat conical shape. The apparent turning of the plane of oscillation for different points on the surface of the earth is given in the table annexed. Wherever observations have been made, calculations and experiments (as has been already said) have always been found to agree, allowing for inaccuracies of observation.

Geographical Latitude.	Rotation of Plane of Oscillation.		A complete circle is described in
	In 1 minute.	1 hour.	
0°	0°	0°	never.
10°	0° 2·6′	2° 36·3′	5 days 22 hrs. 49·2 min.
20°	0° 5·1′	5° 7·8′	2 ,, 22 ,, 10·3 ,,
30°	0° 7·5′	7° 30·0′	2 ,, 0 ,, 0 ,,
40°	0° 9·6′	9° 38·5′	1 ,, 13 ,, 20·2 ,,
50°	0° 11·5′	11° 29·4′	1 ,, 7 ,, 20·3 ,,
60°	0° 13·0′	12° 59·4′	1 ,, 3 ,, 47·4 ,,
70°	0° 14·1′	14° 5·7′	1 ,, 1 ,, 32·4 ,,
80°	0° 14·8′	14° 46·3′	1 ,, 1 ,, 2·8 ,,
90°	0° 15·0′	15° 0·0′	1 ,, 0 ,, 0 ,,

We see from this table that with a sufficiently long pendulum, the deviation of the plane of oscillation will be noticeable at London and other places of mean and greater latitudes after a few minutes' observation.

Different Direction of the Vertical in Space during Rotation.—In order clearly to understand the daily rotation of the earth, as well as all other motions in space, we must first fully realise the importance of the objection, that if we accept the rotation of the earth as a fact we must occupy after 12 hours the place of our antipodes, or make an angle with our present position increasing in size as we live nearer to the equator. At the equator itself our position becomes exactly reversed; we stand then as if at home we were standing on our heads. An inhabitant of London, for example (Fig. 16), stands at 6 P.M. so that with his position at 6 A.M. he describes

an angle of 102 degrees. This is nothing new, for even with a motionless earth we came on our journey from London in the neighbourhood of Alaska, into a similar predicament. The fact that we do not notice it has already been discussed, when speaking of the spherical form of the earth; but if we were suspended by our feet, we should know it soon enough, even though we were blindfolded. We should feel the blood rushing to our head straight downward like the falling stone. The conclusion therefore is, that at all points of the earth the direction of the zenith is for us "top," and the direction of the centre of the earth "bottom"; at all points of the earth all matter (our blood included) is drawn or pressed towards the centre of the earth. We have learned that by reason of the power of inertia. A stone which we put down without pushing it forward ought to lie still in the place where we put it. But the stone falls to the ground; there must therefore be some external force which imparts this motion, either pressure from above or attraction from below. Since we can find no evidence of a pressure from above, the zenith varying every moment, we must conclude that this force lies in the centre of the earth, the direction towards which all falling bodies move. A plumb-line, *i.e.* a piece of string stretched by a cylindrical weight, gives us the direction of all falling bodies, and if this line were prolonged, it would finally reach the centre of the earth. It is true that two plumb-lines hanging close together appear to be parallel, but this is only apparently so; they would intersect each other at a distance of 4000 miles; thus their mutual inclination is not perceptible.

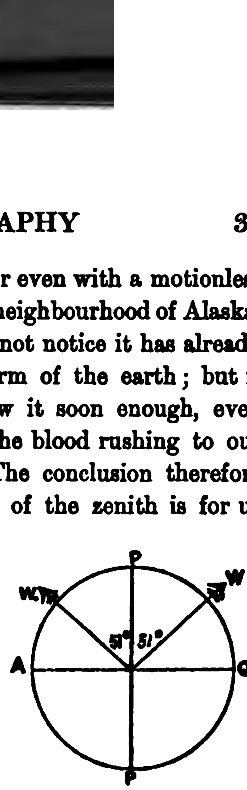


FIG. 16.

The Seat of Gravity.—For the seat of the force which makes bodies fall to the ground we have to accept the centre of the earth. We call this force *gravity* or *gravitation*. It would not be impossible to imagine some mysterious being stationed there, from whom proceeded the power which we call gravity; but it is also possible to imagine that this power proceeds from the centre of the earth without the power itself being actually

stationed there, but being merely the resultant of forces uniting in the central point. If we accept that every particle of matter on the earth (which for our present purpose we will consider to be a perfect sphere, composed of perfectly evenly distributed particles of uniform weight and volume) exerts its attraction on a falling body, this body must fall in the direction of the centre of the earth. For if we connect the falling body A (Fig. 17) with the centre O, any particle of matter X corresponds to Y, lying symmetrically on the other side of the line AO, and their power of attraction, AX and AY, must act in unison with their corresponding attraction in the earth's centre. It is much simpler, however, to accept that the power of gravity, which belongs to the entire mass of particles in common, is a mysterious something in the geometrical centre of the earth.

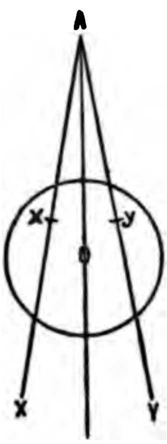


FIG. 17.

All Bodies have Weight.—The properties of gravity can, of course, only be learned by experience and experiment. We ask:—

1. Does the power of gravity act upon all bodies? Have all bodies weight?

2. Does it act uniformly upon all bodies, or are some bodies attracted more, others less?

3. Does it act in the same degree upon bodies at a distance from the centre of the earth as upon those nearer to it, and if not, what is the proportion?

With regard to the first point, we can easily convince ourselves that weight is a universal property, belonging to all bodies. We see that all bodies, without exception, fall to the ground. It is true that under certain circumstances some bodies seem to make an exception to this rule. A piece of paper, a kite, a balloon, do not always fall vertically to the ground; they often move hither and thither, hang suspended in air, or even rise. But in all cases of this sort we can point out the counteracting forces, and finally all bodies do fall to the ground if free to do so.

PHENOMENA AT DIFFERENT LATITUDES

Horizon of Paris, Latitude 48½°.—Our journey now lies to the south. We are already aware that a slight change of station, such as can be obtained by a few hours' walking, makes no perceptible difference in the appearance of the sky, at least not to the naked eye. We must, therefore, take up our station at a considerable distance, and will imagine ourselves at Paris.

We notice that the stars and constellations here are exactly the same as at home, and that they occupy the same relative position towards each other. Even in their position as regards the horizon at Paris the naked eye will scarcely notice any difference as compared to London. There, as here, the pole-star occupies a point rather less than midway between zenith and horizon; there, as here, the sun rises at any given place to our right, at a right angle, above the plane of the horizon, and so on. But if we look a little closer into it, we notice several differences.

As before for London, we must begin by finding the meridian of Paris by means of the gnomon, and thereupon determine its latitude. We find that the latitude of Paris is about 3° less than that of London; the pole-star is about six full-moon lengths nearer to the north point of the horizon, or that much lower, than in London; the latitude is only 48½°. Beyond these little differences the celestial sphere moves in both places round one and the same axis; in both places the sun rises in the east point on March 21 and September 23, and sets in the west point; in both places the rising and setting points lie most to the north on June 21,

and most to the south on December 21; but the east and west amplitudes are on those dates somewhat less at Paris than at London. The rising and setting points are thus somewhat nearer to the east or west point of the horizon.

The day arcs have also undergone a change; the diurnal arc of June 21 is somewhat shorter, that of December 21 somewhat longer, in Paris than in London. The same thing applies to the intervening space of time: the days of the summer months are somewhat shorter, and those of the winter months somewhat longer, than the corresponding days at London. On the other hand, the culminations of the sun are in Paris 3° higher than in London on the corresponding days.

It is evident that in so far as the temperature of Paris and of London depends upon the sun, *i.e.* irrespective of terrestrial influences (natural condition of the soil, water, wind, etc.) and the modifications thereby produced, the day in Paris must be warmer than the corresponding day in London, because the rays of the sun fall at a greater angle. These observations prove that the phenomena of the celestial sphere have suffered no alteration, but merely that the position of the horizon at Paris varies slightly from that at London. If our celestial globe be set for London it can be arranged for Paris by turning the globe on the east-west line 3° to north; *i.e.* we hold the globe steady (in the frame) at the east and west points, and gently pull the pole 3° lower down.

Horizon of Southern Extremity of Europe, Latitude 36°.—At the most southerly points of Europe, somewhere about Cape Matapan, Cape Tarifa, etc., we register a latitude of 36°; the altitude of the celestial equator therefore is 54° (Fig. 18). As compared with London, the difference in the appearances here is much greater than it was at Paris. When looking at the sky we notice at once without any measurements that the pole-star is much lower down. Of the seven stars of the Plough (Great Bear), the star in the tip of the tail (η of the Great Bear) is no longer a circumpolar star, and in the south, stars appear above the horizon which can never be seen in London; amongst them there is one star of first magnitude (Canopus in Argo, the ship). As in Paris, but

in a greater degree, the east and west solar amplitudes become less; the extreme length of the diurnal arc decreases, and the culmination height is greater than in London. On March 21 the sun culminates there at an altitude of 54°. On June 21 the sun stands $54+23=77°$, therefore only 13° from the zenith; even on December 21 its height at noon is $54-23=31°$ above the south point.

A globe set for London can be arranged for Cape Matapan in the same way as for Paris; only instead of turning it 3°, it should be turned $15\frac{1}{2}°$.

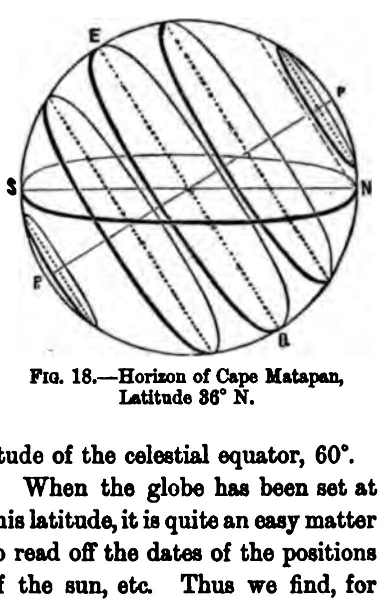


FIG. 18.—Horizon of Cape Matapan, Latitude 36° N.

Horizon of Suez, Latitude 30°.—In Suez (Fig. 19) the latitude is 30°; altitude of the celestial equator, 60°.

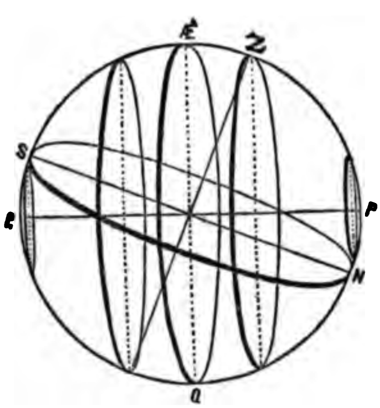


FIG. 19.—Horizon of Suez, Latitude 30° N.

When the globe has been set at this latitude, it is quite an easy matter to read off the dates of the positions of the sun, etc. Thus we find, for instance, that on June 21, at noon, the sun is only 7° off zenith. We leave all further deductions to the intelligent reader.

Horizon of Assouan, Latitude $23\frac{1}{2}°$.—We now come to Assouan, ancient Syene. There the pole-star is only $23\frac{1}{2}°$, about 47 full-moon lengths, above the north point (Fig. 20). The Great Bear and Cassiopeia are no longer circumpolar constellations, but, on the other hand, in the south several new stars and constellations make their appearance, amongst which is the famous Southern Cross.

On March 21 the sun rises there, as with us, in the east point, and sets in the west point, but culminates at an altitude of $66\frac{1}{2}°$, higher, therefore than he does in London on June 21. The successive rising points move northward, and the days become longer; both, however, less perceptibly than at London. On June 21 the east amplitude is only about $25\frac{3}{4}°$, the length of day about 13 hours 23 mins.; but the culmination height, on the contrary, is $66\frac{1}{2}+23\frac{1}{2}=90°$, *i.e.* the sun culminates in zenith. The shadow of a vertical object falls straight under it. At noon

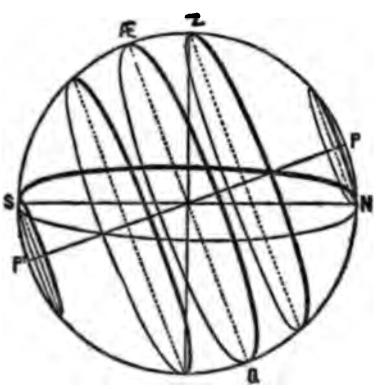


FIG. 20.—Horizon of Assouan (Egypt), Latitude $23\frac{1}{2}°$ N. Here the sun is vertical at noon on June 21.

on June 21 the inhabitants of Assouan have no shadow, and at that hour an open well or pit reflects the face of the sun. After that the rising points gradually go back towards the east point, where the sun rises on September 23, and move back still farther south until on December 21 the east amplitude of $25\frac{3}{4}°$ is reached. The meridian altitudes decrease, but even on December 21 the sun still culminates at $66\frac{1}{2}-23\frac{1}{2}=43°$, higher, therefore, than it does at London on March 21, the first day of spring. Consequently in Assouan the heat of the sun on December 21 (the day on which its power is least) must be still greater than in London on the first day of spring; and on June 21 the heat of the sun reaches its

maximum. A globe set for London can be altered for Assouan by turning it nearly 28° on the east-west line.

Horizon at 20° *Latitude* (*Nubia*).—About three or four days' journey south of Assouan (in Nubia, somewhere near the third Nile cataract), we reach a spot with a latitude of 20° (Fig. 21). There the 20th parallel circle, in which, as we know, the sun is on May 21, passes through the zenith. The sun, therefore, culminates there on May 21 in the zenith, and after May 21 he culminates on the northern side of the zenith. On June 21

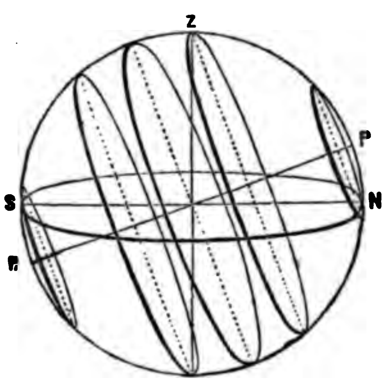


FIG. 21.—Horizon at 20° Latitude (Nubia).

the sun's culmination point is $3\frac{1}{2}°$ (seven full-moon lengths) from zenith to north, *i.e.* $86\frac{1}{2}°$ beyond the south point. From there it retrogrades until on June 21 it once more stands at the zenith. After June 21 the sun culminates from the zenith southward, and on December 21 reaches the lowest height, $43\frac{1}{2}°$ from the zenith ($46\frac{1}{2}°$ altitude), and from that date until May 21 again ascends towards the zenith. For the inhabitants of those regions, therefore, the midday shadow does not fall in the same direction (the north) all the year round, but from May 21 to July 21 it falls to the south. On those two dates they cast no shadow at all at noon, and during the remainder of the year the shadow falls to the north. The longest day (June 21) has only 13 hours 13 minutes, and the greatest east amplitude is 25°.

Horizon of Lake Tzana, Latitude 12°.—Still farther south, in the neighbourhood of Lake Tzana in the Abyssinian mountains, the latitude is only 12° (Fig. 22). At Lake Tzana, therefore, the sun culminates in the zenith on April 21 and August 21, as on those dates it is in the 12th parallel circle, which then passes through the zenith. From April 21 to August 21 the midday shadow falls to the south, and in other respects the phenomena are the same as those of the former station. The sun's meridian altitude on June 21 is 11½° from the zenith to north, 78½° beyond the north point

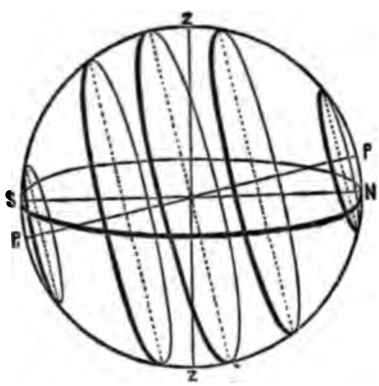


FIG. 22.—Horizon of Lake Tzana, Latitude 12° N.

on December 21, 35½° from the zenith to south, 54½° beyond south point. Longest day, 12 hours 52 minutes; greatest east amplitude, 24°.

We draw attention to the fact that in this latitude the sun (as well as the stars) rises almost perpendicularly to our right (as we face the east) at an angle of 78°.

Horizon of the Sources of the Nile, Latitude 0°.—We now come to the lake district of the sources of the Nile—Lakes Albert Nyanza and Ukerewe (Fig. 23). Here the pole-star is in the north point of the horizon; the latitude is 0°; the equator passes through the zenith; the axis is horizontal; the south pole, not marked by any prominent star, is in the south point of the horizon.

Sun and stars no longer rise obliquely, but vertically above the horizon. East and west amplitudes are equal to their corresponding declinations; all diurnal circles are divided by the horizon into two equal halves; their day and night arcs are all alike (180°). There are no circumpolar stars; every star—the sun included—remains throughout the year twelve hours above and twelve hours below the horizon daily, but in the north and south points the two poles remain motionless, the north pole being known by the close vicinity of our familiar pole-star. On March 21 the sun rises in the east point,

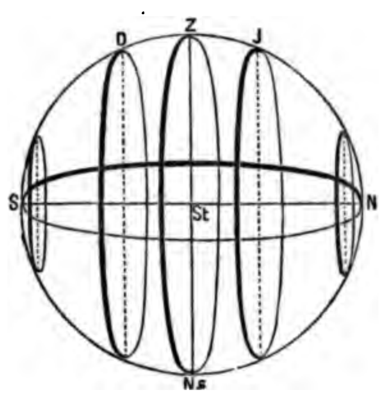


FIG. 28.—Horizon of the Sources of the Nile, Latitude 0°.

culminates at the zenith, and sets in the west point of the horizon. From thence it culminates during our six summer months from zenith towards the north, and during the winter months from the zenith towards the south. It reaches its greatest distance from the zenith (23½°) on June 21 and December 21—on the former date 66½° beyond the north point, on the latter an equal distance beyond the south point. Apart from other meteorological influences, the 21st of March and 23rd of September must be the hottest there, the 21st of June and 21st of December the coldest days, and the two latter, in spite of their different situations, must be equally hot, because the sun reaches the same meridian altitude. For

[redacted]

those regions, therefore, the year is generally divided into two equal halves, in accordance with the influence of the sun, but it is evident there can be no question of seasons in the ordinary sense of the word.

Journey to the North.—We return to London, and from there travel north. Manchester is our first station, but as there is only 2° difference between the latitude there and at London, even the telescope can detect no great differences; the latitude of Edinburgh is 4½° greater than that of London (56°), and the trained eye can, without the help of instruments,

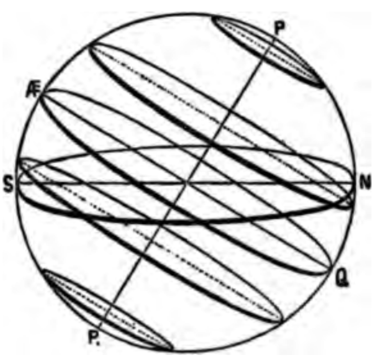


Fig. 24.—Horizon of St. Petersburg, Latitude 60° N.

detect some slight differences, the pole-star being there about eight full-moon breadths higher up in the sky. But we must leave the reader to carry out his own observations from the above-named places, and we proceed at once in imagination to a much more northerly station.

Horizon of St. Petersburg, Latitude 60° N.—In St. Petersburg, and as nearly as possible in Stockholm also, the latitude is 60°, altitude of the equator therefore 30° (Fig. 24). Dividing the quadrant between the north point and zenith into three equal parts, we find that the pole-star stands in the second section. From the 30th parallel all the stars are circumpolar stars, as, for instance—Lyra, Perseus, the greater part of Andromeda and Boötes.

On March 21 the sun rises in the east point, ascends the same as on

all other days, and the same as the stars, at an angle of 30° to our right; culminates at an altitude of 30°, and sets in the west point.

The rising points recede from the east point more rapidly than they do with us, and the days increase in length more perceptibly. On April 21 the declination of the sun is 12°. Its east amplitude, however, reaches on that date $24\frac{1}{2}$°, and the length of day is 14 hours 50 minutes. The sun culminates at an altitude of $30° + 12° = 42°$. On May 21 its east amplitude is 43°, the length of day 17 hours 13 minutes, its meridian altitude $30° + 20° = 50°$. On June 21 its east amplitude is 53°, length of day 18 hours 20 minutes, culmination height only $30° + 23\frac{1}{2}° = 53\frac{1}{2}°$, about the same as at Vienna on April 25.

It is superfluous further to detail these simple appearances; it therefore only remains to remark that on December 21 the length of day is 5 hours 40 minutes, and the sun culminates at an altitude of $30° - 23\frac{1}{2}° = 6\frac{1}{2}°$. Because of this very low position of the sun a gnomon of not more than 1 yard in length will cast a shadow of $8\frac{3}{4}$ yards, or in other words, behind a wall 1 yard high, no direct ray of the sun could reach the ground except at a distance of $8\frac{3}{4}$ yards at noon on December 21. A space $8\frac{3}{4}$ yards square, surrounded by a wall 1 yard high and open from the top, would on that day receive no direct ray of the sun. (In order that a space of the above given dimensions may receive no direct sunlight throughout the year, the wall surrounding it would have to be about 12 yards high.)

Horizon of Tornea, Latitude $66\frac{1}{2}$° N.—We now proceed to Tornea, at the mouth of the river of that name, on the Gulf of Bothnia (Fig. 25). There, or rather a little more to the north, the latitude is $66\frac{1}{2}$°, altitude of the equator $23\frac{1}{2}$°. The region of circumpolar stars has extended; the tropic of Cancer is now its limit (boundary circle); Castor and Pollux, the two bright stars in Gemini, are here circumpolar stars. On March 21 the sun rises in the east point, culminates at an altitude of $23\frac{1}{2}$°, and sets in the west point. Day and night are of equal length. But as early as April 21 the east amplitude is $31\frac{1}{2}$°, and the length of day nearly 16 hours. On May 21 the east amplitude is 59°, and the

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length of day 19 hours 35 minutes. On June 21 the sun is in the tropic (of Cancer) which we have seen is for Tornea the boundary circle of the circumpolar stars. On June 21, therefore, the sun is a circumpolar star, is at midnight in the north point (also very nearly its rising point), at 6 A.M. in the east, culminates at noon $23\frac{1}{2}° + 23\frac{1}{2}°$ beyond the south point; at 6 P.M. the sun is in the west, and at midnight once more at the north point.

After June 21 the points of rising gradually move towards the east point,

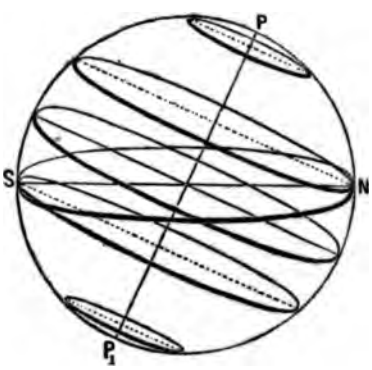


FIG. 25.—Horizon of Tornea, Latitude $66\frac{1}{2}°$ N.

which is reached on September 23, on which date day and night are once more equal. The days now shorten rapidly, until on December 21 the sun enters the tropic of Capricorn, which, for Tornea, is the boundary circle of the southern circumpolar stars. The sun touches the south point only at midday; the night is 24 hours long. The imposing sight of the midnight sun at the time of the summer solstice, which may be witnessed from Mount Asvasaxa, near Tornea, attracts many tourists to that place.

Horizon of Altengaard, Latitude 70° N.—Near Altengaard in Norway, which can boast of an institute of physical science and a magnetic observatory, the latitude is nearly 70°; at Hammerfest, the most northerly town of Europe, a little over 70° (about $70\frac{1}{2}°$). The altitude of

the equator is therefore only 20°, and the 20th parallel, in which the sun is on May 21, is the boundary circle of circumpolar stars (Fig. 26).

On May 21 the sun reaches the north point at midnight, culminates at noon 40° above the south point, occupies each day at midnight a somewhat higher position than the previous day, while it makes 30 complete circuits round the horizon; and on June 21, at noon, reaches an altitude of $43\frac{1}{2}°$ above the south point (upper culmination), at midnight $3\frac{1}{2}°$, or seven full-moon lengths, above the north point (lower culmination). Then it describes

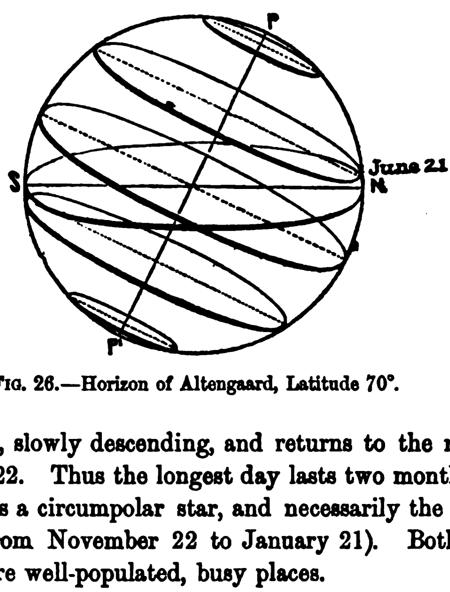


Fig. 26.—Horizon of Altengaard, Latitude 70°.

another 30 circuits, slowly descending, and returns to the north point at midnight on July 22. Thus the longest day lasts two months, *i.e.* during that time the sun is a circumpolar star, and necessarily the longest night is equally long (from November 22 to January 21). Both Altengaard and Hammerfest are well-populated, busy places.

Still more Northerly Places.—From the above stations it is extremely difficult to penetrate farther north, but, frequent whaling and sealing expeditions are made during the short summer in the Arctic Ocean, and the ships sail into regions of 75° or even 80° latitude. The North Polar expeditions penetrated farther north even than that; the Austro-Hungarian expedition reached a point above 82° latitude, and Nansen and Peary higher still.

So far no one has succeeded in reaching a higher northern altitude, but considering all the above calculations, there is no doubt that, if only we could travel far enough north, a point would be reached where the pole is in the zenith. That would give us the vertical position of the axis (latitude 90° parallel sphere); not one star would rise or set; there would be only circumpolar stars, their boundary circle coinciding with the horizon (Fig. 27). The celestial sphere would move from left to right like the hands of the clock, and with it the sun, spirally ascending from March 21

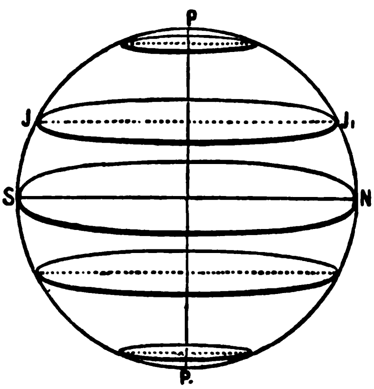


FIG. 27.—Horizon with Vertical Axis, Latitude 90°.

to June 21, and descending from June 21 to September 23, would be for half a year (our summer months) a visible circumpolar star, and then disappear and become an invisible circumpolar star for the remainder of the year. We do not relinquish the hope that this point may yet be reached, notwithstanding the enormous sacrifices of life and money which these expeditions have already demanded.

Horizons with Southern Latitudes.—We now return to the torrid zone and the neighbourhood of Lake Ukerewe, where the axis of the celestial globe occupies a horizontal position, and continue our journey from there towards the south. Our pole-star is the first to disappear; it sinks

behind the horizon in the north point, but in the south, *i.e.* to our right as we face the sunrise, where up to now the sun culminated, a new point arises which does not change its position above the horizon, but unfortunately this point is not marked by any specially brilliant star; it is a new pole, the south pole. The sun rises here on the same side as it did above the horizons we have just visited, but, turning our face towards the rising sun, we see it ascend no longer obliquely to our right, but obliquely to our left (when journeying south the sun always rises to our left, when journeying north always to our right). The sun's culmination points here (and henceforth) lie on the side of the meridian circle facing north. The southern portion of Lake Nyanza has S. latitude 12°, altitude of the equator 78°. On October 22 the sun culminates in the zenith, after that date from the zenith southward, reaching its lowest point on December 21, 11½° from zenith to south, 78½° beyond south point. Then the culmination heights increase again, until at noon on February 19 the sun is once more in the zenith. From that date they sink on the north side; on March 21 the culmination height is 78° beyond the north point, and on June 21 the minimum is reached with 54½° (shortest, coldest day). All the appearances correspond to those of Lake Tzana, 12° N. latitude, only that right and left, summer and winter, are reversed. Sofala has 20° S. latitude; the sun passes through the zenith on November 22 and January 21, culminates on June 21, 3½° from zenith towards the south, on December 21, 43½° from zenith towards the north. The appearances are analogous to those of Nubia. At Cape Corrientes, on the African coast of the channel of Mozambique, the S. latitude is 23½°; there the sun passes through the zenith only once a year, on December 21. The appearances here are a counterpart to those of Assouan (Fig. 20). In Port Natal, 30° S. latitude, the sun culminates all the year round on the northern half of the meridian circle; the appearances are the same as at Suez. At the Cape of Good Hope, with 35° S. latitude, the phenomena are almost analogous to those of the extreme south part of Europe.

Taking ship at the Cape of Good Hope, and sailing from there south-

ward, S. latitude becomes greater, and we reach spots which form counterparts to Paris, London, St. Petersburg, and lastly Tornea (Fig. 25). The station corresponding to Tornea is very difficult of access; there the sun is on December 21 at midnight in the south point, ascends at an angle of 23½° to the left (east), reaches the east point at 6 A.M., culminates at noon, 47° beyond the north point, then begins to descend, is in the west at 6 P.M., and at midnight returns to the south point. On June 21 the sun never rises, but only touches the north point.

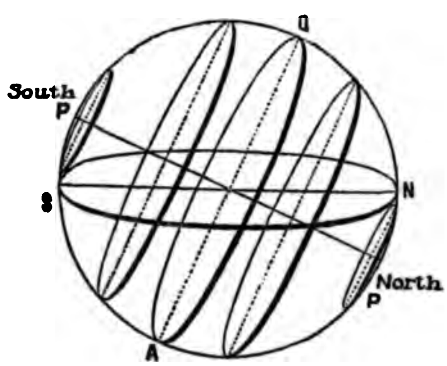


FIG. 28.—Horizon at Cape Corrientes, Latitude 23½ S.

The Diminution of Gravity inversely as the Square of the Distance.—When we attribute the falling of bodies to the collective power of attraction of all particles of matter, we can express this equally well by saying that this is the result of a force proceeding from the central point, and acting with the combined force of all the individual attractions. But it stands to reason that this force, like any other radiating in different directions from one central point, must act inversely as the square of their distance from each other. We will imagine a hollow globe, with any given semi-diameter, round this central point; the action of the attractive force will be diffused over the entire surface of the globe. When we take the semi-diameter two, three, four times larger, the surface of the globe becomes four, nine,

sixteen times larger. The action is thus spread over a space four, nine, sixteen times as great; on an equally large space, therefore, falls only a fourth, a ninth, or a sixteenth part of the action.

This statement, which reason tells us must be correct, has to be proved by experiment. If it be correct, then a body at twice the distance must have only a quarter of the original velocity, and act with only a quarter of the pressure. As we have to exclude direct experiment, and also the pendulum experiment specially adapted to this purpose, there is nothing left but to find out whether the pressure, the absolute weight of a body, diminishes with its distance from the earth. But how can we do this? Avoirdupois weight cannot help us, for if a body exercises less pressure on the top of a mountain than at the bottom (sea-level), *i.e.* has less weight, it stands to reason that the scales with which we weighed the body must have undergone the same diminution of weight. We must, therefore, endeavour to measure the amount of attraction or pressure exercised by the gravity of the earth, by taking as a standard measure some force not influenced by the gravity of the earth. There are many ways in which we might ascertain the general diminution of gravity, if we could remove ourselves sufficiently far from the surface of the earth. If, for instance, we could ascend as high as 4000 miles above the surface, we ought to be able to lift four times as heavy a weight, because at that elevation 4 lb. would only exercise the attraction or pressure of 1 lb. here. But supposing we ascended a mountain one mile in height, the weight of a body there would stand in relation to a body at sea-level as $4000^2 : 4001^2$, *i.e.* as $4000 : 4002$ (nearly), or $1 : 1{\cdot}0005$.

Elasticity is the most satisfactory medium to verify the diminution of weight, the principle of the spring-balance.

A spiral spring is fixed in the centre of a vertical board (Fig. 29). To the other end of the spring is fastened a needle and a little dish to hold the weights. The needle plays up and down the scale as the spring is being more or less stretched.

With the assistance of this balance it is possible to determine the

weight of different bodies (always, of course, within the range of the balance). Letter-scales are generally made upon this principle. If 100 g. be laid on the spring-balance at sea-level, the needle will register 100 on the scale, but at the top of a mountain one mile high (say in the Himalayas) 100 g. will only register 99·7 g.

Bodies have Mutual Attraction.—All matter is attracted by the earth. But if we regard the power of attraction as a force which belongs alike to all particles of matter, we come to the paradoxical conclusion that the stone which we lifted from the ground and dropped again, so long as it formed part of the earth, did attract, but when lifted and let fall was being attracted. Continuing this train of thought, we begin to suspect that perhaps attraction is a reciprocal force, that all matter has mutual attraction, and that the stone falls to the ground simply because its mass is immeasurably small as compared to that of the earth. It is somewhat similar to our adding another particle of dust to an already heavily weighted balance. Be the balance ever so sensitive, it would cause no perceptible difference; while if the scales were unloaded the particle of dust would be sufficient to destroy the balance of the scales. It will therefore be desirable to prove in some way that the power of attraction is a force resident alike in all particles of matter.

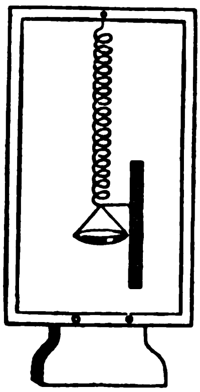


FIG. 29.—Spring-Balance.

Cavendish's Torsion Balance.—If we suspend a small ball to a thin piece of wire, the ball will pull the wire in the direction of the centre of the earth. If we were able to bring a sufficiently large mass to bear upon it sideways, the ball would be deflected to the side. But we have no such mass at our disposal large enough to counteract the attraction of the earth. If, however, we could make the ball irresponsive to the attraction of the earth, a ball of moderate size would be sufficient to attract the smaller ball. This is the principle of the Cavendish Torsion balance. A light rod is horizontally suspended, fastened in the centre to a fine long

vertical wire without torsion (say of platinum) (Fig. 30). At each end of the rod is a small ball, while to the right and left of the point of suspension of the lever are two metal balls of medium size, say of iron or lead, so as not to choose the more expensive platinum or gold. When the lever coincides with the line connecting the centres of the two larger balls, it will remain in repose, because both balls attract the rod towards their own centre. But when we place the lever in the dotted position the little ball *a* is nearer to *A* than to *B*, and *b* is nearer to *B* than to *A*. Thus, because their power of attraction increases in proportion as the inverse square of their distances, *a* is attracted with more force towards *A*, and *b* towards *B*, and the rod twists in the direction of the arrows. If we replace the balls *A* and *B* by glass globes filled with mercury, the result will be the same.

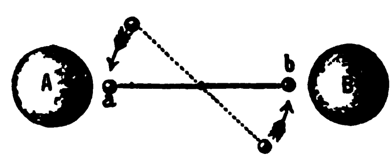


FIG. 30.—Cavendish Experiment.

These experiments have been made several times, and confirm the universal attraction of matter. We must remember, however, that these experiments were not made to establish the fact of universal attraction, of which fact one was already fully convinced, but they were made in order that, by comparing the magnitude of this attraction with that of the earth, the weight (mass) of the earth, and hence its density, might be ascertained, but this is not of present interest to us.

Deflection of the Plumb-Line on Isolated Mountains.—Another method by which to prove the universal attraction of matter is found in the following suggestion. In the neighbourhood of an isolated mountain a pendulum ought to be deflected towards the mountain, and away from the direction of the centre of the earth—only, of course, in the immediate neighbourhood of the mountain; for as the force of attraction varies as the inverse square of the distance, the deflection must become imperceptible at a moderate distance. How can this experiment be made? For the deflection must affect everything, our own position included. Even our zenith will be displaced without our being aware of it. At first sight we might be disposed

to think that the pendulum in the vicinity of such a mountain must form with the horizontal, *i.e.* the surface of still water, an obtuse angle towards the mountain and an acute angle in the opposite direction. But if we keep in mind that the water-level, the horizontal position, is also the result of attraction, it becomes evident that in this, as in all cases, the vertical and the horizontal must make a right angle with one another. The following experiment, which also was primarily made to ascertain the density of the earth, will supply us with the desired result.

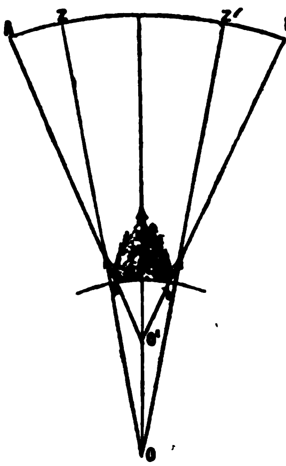


Fig. 31.

From two opposite stations due north and south of an isolated mountain (the experiment was made in 1772 on Mount Schiehallion in Scotland), we determine the zenith distance of the same star at the time of its culmination (Fig. 31). The difference of these zenith distances (as the star occupies the same point in the heavens) gives the distance between the zeniths of the two stations, *i.e.* the difference of the points in the heavens towards which the pendulum points at either place, *i.e.* the arc AB in the heavens. If the mountain does not deflect the pendulum, their relative lines of prolongation must meet in the centre of the earth, and the arc ab on the surface of the earth must number as many degrees, minutes, and seconds as AB, *i.e.* as the angle at the centre of the earth. We measure the actual length of the arc ab. That this can be done, notwithstanding the intervening mountain, has been proved effectually by the engineering work done during the construction of railways, etc. From the length of this arc we can calculate the number of degrees, minutes, and seconds of ab, for we know that one degree on the earth's surface measures 60 geographical miles or 69·2 English statute miles, or more exactly, we know the length of degree of the meridian at any place. If this length corresponds with the length as

determined by astronomical observation, then there is no deflection, but if there is any deflection the two pendulums will make a greater angle than the plumb-lines from the two extreme points of the arc *ab* with the central point; the geodetical arc *ab*, expressed in degrees, must be smaller than the astronomical corresponding arc *AB*. (This is merely illustrative of the principle since the earth is not exactly spherical.)

The Law of Universal Attraction.—It is quite evident from the foregoing experiments that the power of gravity is a force belonging alike to all particles of matter. Every particle of matter in the universe attracts every other particle with a force inversely as the square of its distance from any such particle. If one body is considerably larger than the other body, as is the case with the earth and a falling stone, or even a large rock, we ignore the attractive force of the smaller body, and simply say that it is attracted. Every body is attracted in proportion to the mass of the body attracting it, and in inverse proportion to the square of its distance from the point in which the power of attraction of all separate particles of matter is supposed to be centred (centre of attraction, in the case of the earth, is the centre of the earth). Supposing the earth preserved its original weight (mass), but were so compressed that its semidiameter became reduced to half its original length, all bodies on its surface would fall with four times increased velocity, and a body now weighing one pound, would then weigh four. If, on the other hand, the earth preserved its present bulk, but possessed twice its mass, bodies would fall on to its surface twice as swiftly, and a body now weighing one pound would then weigh two. But if the smaller body were not insignificantly small in comparison with the larger one, the two bodies would gradually approach one another (of course the smaller one travelling quicker than the larger), and the magnitude of their mutual attraction would depend upon the product of the masses of the two bodies, and their distances from each other.

MOTION OF THE EARTH AND OF THE SUN

We may look back with some satisfaction upon the results so far obtained. We have acquired a deeper insight into the phenomena of the universe, and have found their explanation so much simpler, so much more reasonable, than we thought possible before, that we cannot but rejoice in the consciousness that, although we may not yet know all the truth, we are at any rate in a fair way towards knowing it. The fact of the earth moving round its axis with uniform velocity is so absolutely in accordance with the law of inertia that, once accepting the fact of rotation, we could not conceive of it being otherwise. A first cause for this motion, which must exist, we know not. The primary causes and final issues of things are hidden from us, and our deductions and investigations must be based upon observations. In our case we can reason in this way: We know that the earth rotates on its axis; the original cause of this motion is for the present, and possibly for ever, hidden from us, but it is in accordance with all laws of motion, with all existing phenomena; therefore it is reasonable to accept the theory as correct. The only thing which might possibly be raised in objection to it would be that it is hardly possible for us to make, say, a top spin, without at the same time producing a local secondary motion. Our case, however, is somewhat different. On the one hand, we cannot free ourselves from the power of gravity or from the resistance of universal motion; and on the other hand, a progressive movement of the earth would be quite conceivable, without our being aware of it.

Motion of Falling Bodies accelerated by a Side Push.—We should feel a great deal more satisfied if it were possible to reconcile the revolution of the moon and of the sun with the laws of gravity and of inertia as set forth in the preceding section. In order to attempt to do this we must examine somewhat more closely the phenomena attendant upon gravitation; we must take a case where, besides the power of gravity, another force is brought to bear upon a body. We take a stone and drop it, not straight down, but with a slight side push, or we shoot a bullet from a gun in horizontal direction; in both cases these bodies describe a curved line concave towards the earth. The greater the force brought to bear, the less curved will be the line, and the greater the distance traversed before the body touches the earth. This phenomenon is easy to explain. In obedience to the law of inertia, a body will retain the straight direction imparted to it, with unchanging velocity, but in obedience to the attraction of the earth, it is gradually drawn away from this straight path, and the result is a curve. Assuming the push with which it is sent forward ever to increase in strength, a point must be reached when the body does not fall towards the earth any longer, but is forced to revolve round the earth either in a circle or other curve.

Let us see whether the moon's motion could be thus explained. We naturally take the moon's motion first, because the moon is comparatively near to us, at any rate much nearer than the sun. We must be sure of two things: first, that the moon is small enough to justify our disregarding (at any rate in a preliminary approximate calculation) its attraction upon the earth, which must exist, because it is attracted by the earth, and all attraction is mutual. Secondly, we must be able to prove that the deviation from the straight line of motion is sufficiently great to be accounted for by the attraction of the distant earth. The former we can allow, because we know that the earth is more than eighty times as heavy as the moon. The second point we will now inquire into.

Falling of the Moon towards the Earth.—Let MX (Fig. 32) be a portion of the moon's orbit, M, the moon, O, the centre of the earth. If the moon

moved merely farther from the point *M* because of its inertia, it would travel in a straight line in the direction *MY*. If in a unit of time the moon reached *M′*, then *PM′*, which equals *ML*, represents the amount by which during that time it has come nearer to the earth. This space *ML*, therefore, represents the attraction of the earth. Let a second be the unit of time. We can then calculate the piece *MM′* (the path of the moon) in one second. As the diameter of the moon's orbit, *i.e.* its distance from the earth, is 240,000 miles, the circumference of the path must be $2 \times 3{\cdot}14159 \times 240{,}000 = 1{,}500{,}000$ miles nearly. The time required to travel this distance is $27\frac{1}{3}$ days $= 27\frac{1}{3} \times 24 \times 60 \times 60$ seconds $= 2{,}361{,}600$. The moon, therefore, accomplishes in one second a distance of $\frac{1{,}500{,}000}{2{,}361{,}600} = {\cdot}64$ mile.

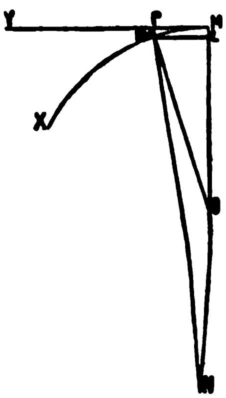


FIG. 32.—Fall of Moon towards the Earth, or deviation from the straight line of motion.

This is the length of the arc *MM*. For all practical purposes this arc may be called a straight line, as it represents the 2,361,600th part of the whole orbit, therefore only $360° \div 2{,}361{,}600 = 1{,}296{,}000'' \div 2{,}361{,}600 = 0{\cdot}56$ seconds of arc. The distance *ML* may be calculated by elementary geometry, either by the use of the isosceles triangle *MM′O*, or the two equal triangles *MM′L* and *MM′N*, in which latter *MN* is a diameter of the moon's orbit (Euclid, Book III.). In both cases we find that the distance *ML* is equal to the square of MM_1, divided by the diameter of the moon's orbit, *i.e.* the square of ·64 divided by twice 240,000 miles. Both numbers must be expressed in the same units. We might also divide the square of 1·019 by twice 383,000 kms., in which case the result (the length of *ML*) would be obtained in kilometres. This comes to $1019^2 : 766{,}000{,}000 = 1{,}038{,}361 : 766{,}000{,}000 = 0{\cdot}00136$ m. $= 1{\cdot}36$ mm. In one second the moon comes about 1·36 mm. nearer to the earth, or each second the moon falls towards the earth through a distance of about 1·36 mm., or $\frac{1}{19}$ in.

It has, however, been proved after careful investigation that a body falls to the earth in one second from a distance of about 16 feet (or 4·9 mm.). As the moon's distance from the earth is about 60¼ earth's semi-diameters, and the force of gravity varies inversely as the square of the distance, it is obvious that the force of attraction is diminished $60\frac{1}{4} \times 60\frac{1}{4} = 3630$ times. A body at that distance, therefore, be it the moon or some other body, is attracted by the earth at such a ratio that in the first second it falls towards the earth $4900 : 3630 = 1{\cdot}35$ mm. These figures, which show with what speed the moon must fall towards the earth in one second according to the accepted laws of gravitation, correspond with the moon's actual velocity, as determined by its motion, to within 0·01 m.—closely enough to admit of no doubt with regard to the assumption that it is the earth which keeps the moon within its orbit. These, moreover, are merely rough figures; a more exact calculation reduces the difference to nil.

Motion as applied to the Sun. It is not the Sun but the Earth that moves.—Of course the same principles could never be applied to the motion of the sun. We have here to deal with a body not merely of larger size than the earth, but of such gigantic proportions that the earth by comparison becomes immeasurably small. We have for the time being taken no notice of the force of the moon's attraction upon the earth, as the moon has only about an eightieth part of the earth's mass. Now, according to its volume the earth is about a millionth part of the sun; if the force of gravity does exist between them, it is not the earth but the sun which exercises the greater attraction. And the more we study the laws of gravitation, the more unlikely it becomes that the earth should be the force which keeps the sun in its place. The insignificant earth cannot possibly be the cause of the sun's motion, and yet the centre of the sun's orbit lies in the centre of the earth, just as in the case of the moon's orbit. Is this another instance of optical illusion? Can it be that the sun's motion is merely apparent, and that the phenomenon must be explained by another motion in which we take part, therefore earth-motion again? Assuming that the distance of the sun from the earth,

in spite of its immensity (93,000,000 miles), is in comparison with the distance of the fixed stars, or as compared to the size of the celestial sphere, so small that a changing of its centre by 93,000,000 miles would not be noticeable, then the annual revolution of the sun from west to east, between the constellations of the zodiac, *could* be explained by a revolution of the earth.

Fig. 33 illustrates this. The large circle represents the ecliptic, and

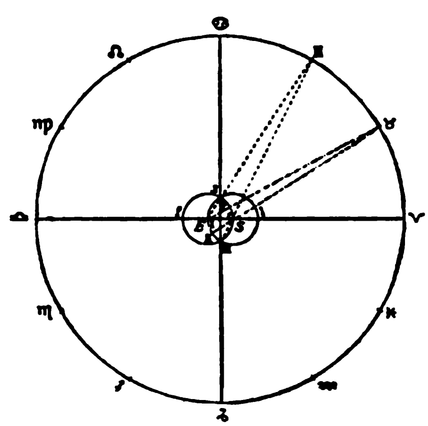


FIG. 33.—Diagram.

the twelve signs of the zodiac are marked on its circumference. We must, however, try to imagine this circle of such immense size that its centre might be either in *S* or in *E*, or in other words, that the line *ES*, although to us it represents a distance of 93,000,000 miles, contracts at that standpoint into one single point. According to this assumption the circle in our diagram is still far too small. We will for the present assume that the sun moves round the earth. Let *E* be the centre of the celestial globe, and the small circle round the centre *E* be the sun. When the sun stands in *S* we see it from *E* in ♈. It will then be March 21; when during the next thirty days the sun travels

from *S* to 2 it moves for the station *E* from ♈ to ♉ in the sky; a further advance to 3 brings the sun from ♉ to ♊, etc., round the whole circle. But if we leave the sun at rest in *S*, and take it for granted that we, with the earth, move from March 21 to April 21, from *E* to II. in the circle round *S*, it must still appear to us as if the sun travelled the distance in the sky between ♈ and ♉; when the earth moves from II. to III. the sun appears to travel from ♉ to ♊, etc.; in short, the phenomena are exactly the same whether the sun revolves round us from west to east or whether we revolve round the sun also from west to east. When the sun, as seen from the earth, stands in Aries, the earth, as seen from the sun, would be in Libra. We could say, therefore, transporting the motion to the heavens: A revolution of the sun through the signs Aries, Taurus, Gemini, etc., beginning from Aries, and a revolution of the earth through the signs Libra, Scorpio, Sagittarius, etc., starting from the point of the autumn solstice, lead to exactly the same phenomena, provided always the sphere be of sufficiently large proportions. Now, as it is impossible to assume that the gigantic mass of the sun should move round the earth, and as we have been convinced more than once that our ideas of the size of the universe have deceived us, we do not hesitate to accept the theory that the annual revolution of the sun, as well as the diurnal, is merely an apparent one, and that the delusion is effected by the earth's revolution round the sun. This theory is further enforced by the fact that the annual revolution of the earth round the sun, and its rotation, have the same direction from west to east.

Position of the Earth in its Orbit.—What should be the position of the earth in its orbit, the ecliptic, in order that all the phenomena, also those relating to the seasons, may be in harmony? As the poles remain stationary during the annual revolution, the axis must also remain in the same position, *i.e.* during the motion the axis remains parallel to itself. But the earth's axis, and consequently also its prolongation, the axis of the celestial sphere, describes in the course of a year the surface of a cylinder; its extremities describe a circle of 93,000,000 miles in diameter, but this circle is contracted into one point, the pole. Its

axis forms an angle of $66\frac{1}{2}°$ with the plane of the ecliptic, as the latter is inclined at an angle of $23\frac{1}{2}°$ to the plane of the equator. On March 21 its inclination is such that the line joining the centres of sun and earth strikes the equator (Fig. 34 at the top of diagram—Spring). On June 21 the earth has advanced 90°, from the first point of Libra to the first point of Capricornus. As, however, the axis has retained the same position (direction) in the space, the line joining the centres of sun and earth now strikes the tropic of Capricorn (Fig. 34 at

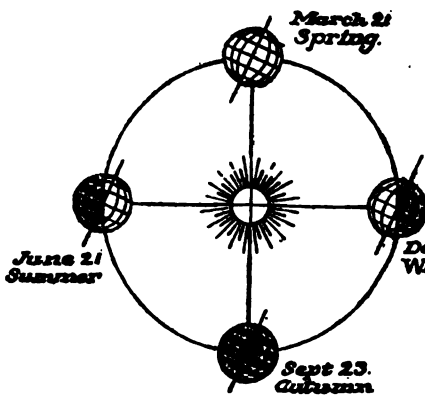


Fig. 34.

left hand of fig.—Summer). On September 23 its position is as indicated in the diagram by the figure at the bottom—Autumn, and on December 21 as in figure at right hand—Winter. It is easy to see that the phenomena at the several points of the earth, as here illustrated, agree with what we know already.

Explanation by Means of a Terrestrial Globe.—In order that this most important fact may be thoroughly understood, we will explain it more fully by using the globe. In this case the horizontal ring of the framework does not represent the horizon, but the plane of the ecliptic. All globes are arranged to serve this double purpose. On the horizontal ring the

names of the signs of the zodiac have been inscribed: at the east point, Aries; at the north point, Cancer; at the west point, Libra; and at the south point, Capricornus. In the spaces between these the other signs follow in their order. We now fix the globe so that the north pole comes $66\frac{1}{2}°$ beyond north point. We place a lamp in the centre of a smooth table, preferably a round one, in such a way that the light falls straight on the extension of the plane of the ring (the ecliptic). The edge of the table is now divided into twelve equal parts, and inscribed with the names of the zodiac. The globe is placed on the table's-edge, on the spot inscribed Libra, so that the line connecting the lamp with the centre of the globe passes through the point in the ring marked Aries. This brings the globe in the right position with regard to the lamp representing the sun; we must imagine the pole-star to lie in the extension of the axis. If we have been careful to arrange the zodiacal signs on the table's-edge according to their position in space, Aries in the eastern quarter, etc., the extension of the axis will point in the position indicated, not actually towards the pole-star, but in a northerly direction towards the meridian.

Representation of the Position on March 21.—The position in the previous paragraph represented the vernal equinox. Our globe, the earth, stands in the sign Libra; the sun, as seen from the globe, in Aries. The sun's rays fall perpendicularly upon the equator; the line of illumination passes through both poles. Turning the globe, we see that over the entire earth day and night are equal, because the line of illumination cuts all the parallel circles in two equal halves.

Representation of the Position on April 21.—We now place the globe in position in the point Scorpio, taking care that the axis remains parallel to itself, and does not change its direction with regard to the room in which we are. As seen from the globe, the lamp representing the sun now stands in Taurus; the position corresponds with April 21. The line joining the flame and the centre of the globe passes through the 12th parallel of north latitude. The line of illumination reaches 12 degrees beyond the north pole, divides only the equator into two equal parts, and the northern parallel circles in such a way that the greater part falls within the range of light,

and the southern parallel circles so that the smaller part falls within the range of light. At a distance of 12 degrees from the north pole, *i.e.* from the 78th parallel of north latitude, all circles are day circles, and in the south all are night circles. Turning the globe, we see all the phenomena as they actually appear upon the earth.

Representation of the Position for the other Parts of the Year.—Again, without altering the direction of the axis, we place the globe in the point Sagittarius, and thus get the position for May 21. A further advance to Capricornus brings us to the summer solstice, June 21, when the line of illumination passes through the polar circle. This may be continued until the globe once more stands in the sign Libra, and the year is completed.

Comparison with the Previous Statement.—It is advisable carefully to compare this circular movement of the earth with the formerly accepted circular movement of the sun. In order to do this the lamp should again be placed in Aries, and the globe, without altering the direction of the axis, in the centre. This is the vernal position. Leaving the globe in the centre, the lamp is moved to Taurus; this is April 21; and so on through all the signs.

CARTOGRAPHY

Definition of Position

The earth is a sphere, or very nearly a sphere, but on close investigation several deviations from the spherical form become apparent. It is therefore of the utmost importance to determine the position of places with such absolute accuracy as to be able to find them again immediately when required. There are several methods by which the earth's surface can be measured and divided, which we will shortly enumerate and compare.

First, there is the ordinary geographical definition of place. The position of a place upon the earth's surface may be determined by simple geographical latitude and longitude. The latitude of a place is its distance from the equator, measured upon the meridian of the place in degrees, minutes, and seconds, either north or south, according as the place is situated either on the north or on the south side of the equator. The longitude of a place is the distance on the equator between any selected first meridian (generally that of Greenwich) and the meridian of the place, either to the east or to the west on the equator. Thus we get north and south latitude, and east and west longitude.

Hipparchus (160-125 B.C.) first applied this method of defining places by latitude and longitude on the terrestrial sphere, and made use of these terms, already familiar under the old system, whereby distances on the supposed disc of the earth (or the inhabited belt of the earth) were determined by longitudinal and latitudinal measurements,

reckoned from the equator. Thus the ancients said the earth was 70,000 stadia long and 40,000 wide.

Secondly, there is the angular definition of place on a plane and on a curved surface. The position of a place on a plane surface is most easily determined by measuring the distance of the point to be determined in relation to two straight lines intersecting each other at right angles.

The perpendicular distance y between a point Q and the horizontal line of intersection is called the *ordinate*; the distance between the point of contact of this line with the horizontal line of intersection, reckoned from point O, or the point where the two axes bisect one another, is the *abscissa*, x. These two lines together form the rectangular *co-ordinates* of the place. Then we get a positive and a negative side, reckoned from the point of bisecting of the two axes or lines of intersection. The two co-ordinate axes, $x = O$, $y = O$, divide the plane into four quadrants.

The equations of the co-ordinate are respectively $\begin{matrix} x = O \text{ (axis of } y) \\ y = O \text{ (axis of } x) \end{matrix}$

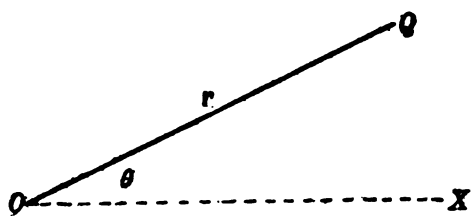
		Y		
(2)	$x -$		$+ x$	(1)
	$y +$		$+ y$	X
	$x -$	O	$+ x$	
(3)	$x -$		$- y$	(4)

In (1) abscissa and ordinate are both positive; in (2) abscissa negative, ordinate positive; in (3) abscissa negative, ordinate negative; in (4) abscissa positive, ordinate negative.

Thus we see that ordinates drawn upwards are positive, and when drawn downwards negative; abscissæ to the right are positive, to the left negative.

Besides this method for defining the position of places on the plane surface, by rectangular co-ordinates, there is the system of polar co-ordinates. According to this system the position of a place is determined when its distance is known from some fixed point, called the pole, and the angle formed by the line drawn from that pole to the point to be

determined, and an initial line or axis. If O be the pole (Fig. 35), OX the polar axis, point Q is defined by the angle θ, and the distance $OQ = r$. The registering and reading of co-ordinates can be simplified by drawing round the pole as centre concentric circles determined by the semi-



Polar co-ordinates r, θ. $OQ=r$. Angle $QOX=\theta$.

FIG. 35.

diameters of 1, 2, 3 . . . units of length; and further, by drawing a number of radii passing through point O, and including all possible angles with OX from 1° to 359°.

The co-ordinates of the plane figure are similar to the spherical co-ordinates of the sphere. A polar co-ordinate system is used; only, instead of straight lines and circles, curved lines are employed, the perpendicular curves radiating rectangularly from the pole to meet the horizontal curved lines drawn round the pole at equal distances from each other.

Thirdly, we have the astronomical definition of place. The above system is not suitable for practical purposes of terrestrial measurements; we therefore look round for an easier method by which absolutely to define the position of a place upon the surface of the earth. This we find in observing the revolution of the earth round its axis, the two poles and the visible bodies in the celestial sphere being our fixed points. With the help of these the earth's surface can be divided into a regular network of lines and curves. The network consists of two groups of circles, called parallels of latitude and meridians of longitude. Mathematical

geography defines parallels as lines of the same latitude, and meridians as lines of the same longitude.

The plane of the parallel which passes through the centre of the earth (and which is therefore a great circle) has latitude 0°, and is the point from which we begin to reckon (equator). We imagine lines of latitude drawn on the earth's surface at one degree distant from each other, thus, 0°, 1°, 2°, 3°, up to 89°, on either side of the equator. This constitutes the first group of lines of the network. Perpendicularly to these are the meridians. A meridian is a great circle passing through the poles of the earth, and is the line connecting two or more places where the culminating point of any given star above the horizon is the same. We take the meridian of Greenwich as the first or zero meridian. We imagine the equator to be divided into 360 equal parts (degrees), so that each imaginary segment as seen from the earth appears at an angle of 1°. These segments may be fixed by any fictitious star.

At the same moment in which places situated on the first meridian see a certain point on the celestial equator pass through the plane of their meridian, the places on the next meridian see the culmination of the next point, those on the third see the third, etc. etc. All places which see the culmination of the same celestial point at the same time are said to be on the same meridian. Astronomers reckon the meridians one way round from 0° to 360°, the 360° being identical with the 0°. For geographical and nautical purposes we count 180 meridians east and west.

These meridians complete the imaginary network drawn over the earth's surface. Given the parallel and the meridian of a place, its position is absolutely fixed, and by referring to the fixed bodies in the celestial sphere we can always find its position immediately. If the earth were a homogeneous sphere, meridians and parallels would form intersecting circles. The meridian planes would cut each other at the axis of the earth, always making angles of one degree with one another, and so also the semi-diameter of the earth, passing through the intersecting point of any meridian and any of the parallel circles, would always make an angle of one degree.

In order to define the exact situation of places on globes and maps, the operation here described is much the simplest. In making the network, the distances between the lines should be enlarged or reduced, according to the scale upon which the representation is based.

General Remarks on Map-Making. Representation of Curved and Plane Surfaces

The art of perspective projection has been called into existence by the necessity of representing the surface of the earth, or portions of it, on a reduced scale and in a handy form, yet at the same time making the representation as perfect a copy of the original as possible, from a geometrical point of view. Now, as the earth is a sphere, and the representation has to be made on a flat surface, the chief difficulty of map-making lies in the reproduction of curves and lines on the flat surface so that they are in exactly the same relative position as on the globe. Since any place on the earth's surface can, as we know, be accurately defined by its geographical latitude and longitude, and can by this network of lines at any time be referred to, the problem is virtually solved when once we know how to convey this network on to the flat surface so as to secure absolute fidelity to the original. When once we know how to draw a meridian and parallel on a flat surface, we know how to draw all lines and circles so as to be absolutely true to their originals on the globe.

Since all representations of large portions of the earth's surface have to be made on a greatly reduced scale, it suffices for all ordinary cartographical purposes to regard the earth as a perfect sphere, and we draw our lineal network accordingly. We assume that geographical and geodetical latitude are identical, or in other words, we take as the geographical latitude of a place the angle formed by the radius of the earth at that place and the plane of the equator.

The simplest and most natural method by which to represent the surface of the earth is the sphere, and to do this we proceed as follows:—

From the centre C of the actual terrestrial sphere, we imagine another concentric sphere drawn on a much-reduced scale, its semi-diameter being r as against R (Fig. 36); and we further imagine that the visual rays are drawn through the corresponding points of the two spheres. The points where these rays intersect the surface of the smaller concentrical sphere are the projections of these points upon its surface. Thus a is the projection of A, b of B. On the small sphere, or our artificial globe, the different points stand in exactly the same relation as on the actual terrestrial sphere, and their distances are in proportion to the natural distances, as the semi-diameter r is in proportion to the earth's semi-diameter R. In fact, as AB and ab are concentric arcs, we get:

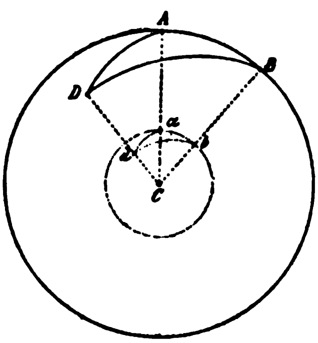


Fig. 36.

$$ab : AB = ac : AC = r : R;$$

hence

$$ab = \frac{r}{R} AB,$$

or, in other words, ab is to AB in the proportion of $\frac{r}{R}$. Projecting a third point D, we see again that

$$ad = \frac{r}{R} AD, ab = \frac{r}{R} DB;$$

therefore

$$ab : ad : db : = AB : AD : DB.$$

Since the corresponding spherical angles of the two spherical triangles ABD and abd are equal to one another, these triangles ABD and abd must necessarily also be equal to one another; hence the figure abd must be similar to the figure ABD.

Scale-Drawing

Every map is a much-reduced representation of the earth's surface, or of a portion of it. The measure of reduction is arbitrary, but should always be determined before the projection is commenced, and every map should have the scale of reduction marked upon it. It should, moreover, be remembered that the measure of reduction always refers to the linear element, never to the area of the map. Thus we see that the globe gives an exact representation of the original sphere in the reduced linear proportion of $r : R$.

Artificial globes are very useful for elementary geographical instruction, but they are insufficient for the more advanced study of mathematical geography. We therefore resort to other methods of representation, and find these in perspective projections on the flat surface.

General Principles of Perspective

By the laws of perspective we represent an object on a flat surface as it appears to us from some given distance. This flat surface, upon which the representation is to be made, is called the plane of projection.

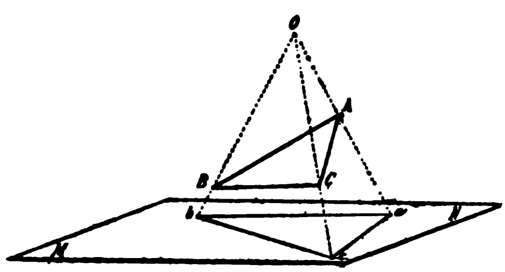


Fig. 37.

We draw from some point of vision O (Fig. 37) three straight lines to the extreme points of a triangle ABC, situated outside the plane of projection, and extending these lines until they touch the plane of projection

MN, the lines of contact between these extreme points of contact with the plane form a second triangle, *abc*, which is the perspective projection of the triangle *ABC*, situated outside the plane of projection. The straight lines *Oa*, *Ob*, *Oc*, drawn from point *O* to the extreme points, are called the *rays of vision* or of *projection*.

In a representation upon the flat surface it should be remembered that a plane and a sphere can only coincide in one point, viz. when the plane is a tangent plane, at the point of contact. The further we are from this point of contact, the greater becomes the distance between sphere and plane, and the greater will be the difference between the representation on the flat surface from its original on the sphere. This fact, that the flat representation is subject to so many distortions and alterations with regard to its original, has led to the idea of drawing the representation on surfaces capable of being developed or rolled out on to a flat surface, and which have not only one point, but a curved line also in common with the sphere. The simplest objects upon which to perform this operation are the cone and the cylinder. Both cone and cylinder can touch a sphere in one circle or intersect it in two circles. Cutting the cone or cylinder along a generating line, it is capable of being developed on the flat surface. In this operation every segment of the once existing circle of contact or intersection preserves its original dimensions; the curve only is altered. The small portion of surface, therefore, bounded by this line will be developed on the flat paper in its proper dimensions. This portion or area will enjoy the same privileges which in projection on the plane of contact can only be enjoyed by the immediate surroundings of the point of contact. As the distance increases from the circle of contact or intersection, the difference between the spherical and cylindrical or conical surface also grows, and the representation made on these bodies is subject to alterations and distortions the same as on the flat surface.

The respective merits of the cone and the cylinder for purposes of projection will be represented when their several developments are being treated of.

ANCIENT CARTOGRAPHY AND LAWS OF PROJECTION

1. *Earliest Attempts at Map-Making*

From the earliest times the want has been felt of representing portions of the earth upon the plane surface, but of these early efforts we have but very vague information. The Greeks were the first to attempt a representation of the whole world (as then known). The earliest on record date from the sixth and fifth eras before Christ, and were executed on brass tablets. The value of these productions, however, should not be overrated: they were ingenious, and called forth the admiration of contemporary writers, but from our point of view they were at best bare linear reproductions (often faulty) of different countries, but not maps in our sense of the word, with accurate representations of different portions of the earth's surface in relative proportion and position. As far as our knowledge goes of these mythical productions, they were always circular, but lacked any mathematical basis or method of projection.

At a later stage, from 320 to 80 B.C., graphic delineations were made of well-known countries, and although more correctly and systematically planned, they lacked all claim to scientific merit.

The ingenuous and unmathematical method of these later attempts we find clearly illustrated in Strabo's Geography (A.D. 15-24). The theory of the spherical form of the earth had by that time been accepted by the scientific world, and Strabo was well aware that it was not so simple a matter to represent a sphere on a plane surface, but he found, as he thought, an easy way out of the difficulty. "It does not greatly matter," he said, "whether meridians and parallel circles are represented by straight lines intersecting one another at right angles, for the sizes and shapes as given on the plane surface give us a very fairly correct idea of what their true proportions must be on the actual sphere. It would, moreover, be superfluous labour to attempt to depict on the plane surface the converging of meridians towards the pole; our imagination

must supplement what the representation lacks in correctness." With this principle as basis for his operations, Strabo imagined all countries stretched out on the flat surface, and in order to define the relative position of places, he employed the method of rectangular co-ordinates.

To represent the two co-ordinate axes he drew two straight lines cutting each other at right angles, one of which represented the parallel of Rhodes, and in the sense of geographical latitude this line divided the then known world into two equal halves. Perpendicular to this line he laid the meridian of Rhodes, which, according to the ancients, also passed through Syene, Alexandria, and Byzantium; in this manner he constructed the co-ordinate system. All other points of the earth he introduced into his plan according to their distance from these principal axes. Strabo knew quite well that this operation could not enable one to define a place very accurately. He therefore explained in writing that these distances should be marked over a large number of meridians and parallels; but in order to do this it was essential to have an accurate knowledge of the relation between latitude and longitude. And this was the chief difficulty in early cartography. True, as early as Hipparchus the *theory* of the methods of astronomical observations and calculations had been brought to a certain degree of perfection; but they could not be practically carried out, partly because the astronomical instruments were inadequate, and partly because verification by observations was virtually impossible. We know of but three or four instances in antiquity in which determinations of the latitude of a place were made by means of the gnomon, and only one instance of a determination of longitude difference—that of Carthage-Arbela.

Failing astronomical methods of definition, they contented themselves with calculating distances, but these are not much use unless the direction is fixed. For the estimation of distances they relied entirely upon the estimates of travellers by land and sea, and the uncertainty of this source of information is proved by the geometrical measurements attempted by Eratosthenes and by Posidonius, which showed a difference of 10,000

stadia. They lacked the instrument by which the direction of the north could be correctly fixed, the magnetic needle.

2. *Projections developed on the Plane*

Cylindrical Projections.—Alexandria, the world-famed town of antiquity, was under the Ptolemies the centre of the exact sciences. The museum, answering to our present-day academies, formed the nucleus of scientific pursuits, and was supported by grants from the royal treasury. Many eminent scholars in mathematics and astronomy emanated from there. Amongst the best known are Eratosthenes, and a few centuries later the astronomer and geographer Claudius Ptolemy (about A.D. 120), whose *Geography* we shall often have occasion to refer to. In the twenty-fourth chapter of the first book of this work he treats of the construction of maps, and the representations on the flat surface occupy the first place. They are based upon the method of cylindrical developments.

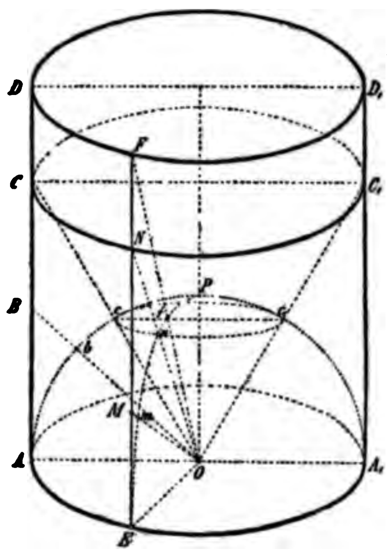


FIG. 38.

The sphere not being a developable surface, a method was conceived by the ancients to convey the delineations of the circular surface on to the surface of some other body, as nearly as possible in character to the sphere, from which it might be developed on to a plane surface. This body was found in the cylinder.

Let APA_1 (Fig. 38) be the terrestrial hemisphere, AA_1 the equator, P the one pole, O the centre of the earth, and AA_1DD_1 a cylinder plane

touching the hemisphere at the equator. We imagine, from the centre O as point of observation, visual rays extending to different points on the surface of the earth, and lengthened to meet the cylinder plane. The points of intersection thus produced are the cylindrical projections of the corresponding points upon the earth's surface. Thus B is the cylindrical projection of b, C of c, etc. All rays of vision drawn towards points on one and the same meridian, as Ob, Oc, are situated on the plane of this meridian, which is perpendicular to the plane of the equator. But the plane of the equator is at the same time the ground-plane of the cylinder; the meridian plane is therefore perpendicular to the base of the cylinder, and cuts through the cylinder axis, which coincides with the axis of the earth. The meridian plane cuts the surface of the cylinder at two right angles, and therefore the rays of vision drawn towards points on the same meridian will produce projections situated in a straight line. Thus AD becomes the projection of AP, A_1D_1 the projection of the opposite meridian A_1P. The projections M and N of points m and n, situated on the meridian PE, will be on the straight line EF, representing the projection of the meridian EP.

Developing the cylinder plane AA_1DD_1, the equator AA_1 will appear as a straight line, while straight lines perpendicular on the equator represent the projections of the meridians. Drawing radii towards different points of one and the same parallel, as, for instance, Oc, On, Oc_1, the points of intersection will all be equidistant from the equator. For

$$\text{Arc } Ac = \text{arc } En = \text{arc } A_1c_1,$$

therefore $$\angle AOC = \angle EON = \angle A_1OC_1.$$

Now the rectangular triangles AOC, EON, and A_1OC_1 have besides the angles AOC, EON, and A_1OC_1 also the lines AO, EO, AO in common, these being the semi-diameters of the same sphere, and therefore the triangles are congruent; hence $AC = EN = A_1C_1$. The same applies to the projections of all other points on the same parallel circle. When unrolling

the cylinder plane, the parallels will appear as straight lines parallel to the equator.

The two sets of lines, therefore, representing meridians and parallels are the groundwork of all map-making. In the plane cylinder development, as we have seen, the net consists of a system of straight lines perpendicular upon each other. Strabo and some of his predecessors projected their maps upon this system; the defects, however, did not remain unnoticed, and several efforts were made to remedy them, although the misrepresentations were not considered of vital importance. It always will seem somewhat incongruous that lines which on the globe are circles should appear as straight lines on the flat surface. Again, the parallel circles on the globe become smaller as they near the pole, but in geometrical cylinder projection they are all equal to one another and equal to the equator. Meridians converge towards the pole on the globe, but in our projection they are all parallel to each other. This causes a distortion in the representation of countries, which becomes the more apparent the greater the geographical latitude of the area delineated. Since the equator preserves its natural dimensions when unrolled on the flat surface, the degrees of longitude remain equidistant, and the equatorial line is therefore divided into 360 equal parts. The degrees of latitude, on the other hand, increase in proportion to the tangent of geographical latitude. Hence[1] $AB = R \tan AOB$, $AC = R \tan AOC$, etc. (Fig. 38). Since $\tan 90° = \infty$ (infinity), the pole cannot be represented in this method of projection, which is at once apparent in the figure, as the ray of vision OP becomes a straight line parallel to the planes of the cylinder. The construction of a scale of latitude of from 5° to 5° would thus involve the multiplication of the semi-diameter of the globe by tan 5°, 10°, 15°, . . . and the result obtained would have to be added to the meridians, beginning at the equator.

To counteract these several defects and misrepresentations, another principle for making the imaginary network of maps was called into existence, which is termed the *equidistant* cylinder projection. Instead of

[1] Once and for all, the radius of a sphere is designated by R.

enlarging the degrees of latitude, they were all made equal to one another, and equal to the equatorial degrees. Thus, drawing the meridians and parallels all equidistant, say, from 5° to 5°, the net will consist of equal squares, and maps thus projected are called *quadrilateral* maps or plans.

Marinus of Tyrus, a geographer only known to us from the *Geography* of Ptolemy, first adopted the method of cylinder projection by equal distances. He lived in the first century of our era, and may be considered the real founder of mathematical cartography. Fully aware of the enormous defects of the quadrilateral system of projection on the normal plane, he tried to obviate the difficulty by altering the plane of projection. For it is evident, since all parallels are made to appear equal in length, the longitudinal distortion of the parallels in higher latitudes, and their respective countries, must be enormous. Marinus, therefore, in his improved system, made the cylinder plane, not a tangent to the equator, but he let it cut the equator at the parallel of 36° N. latitude, so that the base of the cylinder plane corresponded to this parallel circle. Developing the cylinder plane upon this principle, we find that now the parallel circle of 36° N. latitude is reproduced in its natural dimensions. Marinus chose this parallel of 36° because, as mentioned before, it was the parallel of the island of Rhodes, and divided the then known world into two equal halves.

In our days, the cylinder plane is made to intersect the mean parallel circle of the country under projection, viz. the parallel equidistant from the two nearest parallels of the area under projection. In this operation the parallels in the higher latitudes appear somewhat larger, in the lower latitudes somewhat smaller, than in reality; the mean parallel alone appears in its natural proportions. Maps thus projected are called *quadrilateral* in the stricter sense.

The net is made in this way. We draw two straight lines perpendicular to each other, intersecting at the centre; *AB*, *CD* (Fig. 39), being the lines, their centre *O* passes through the centre of the country under projection. *AB* represents the mean latitude parallel, *CD* the mean meridian of the country. Starting from the centre, point *O*,

we mark on CD, upwards and downwards, equal distances, representing the degrees of latitude. R being the semi-diameter of the artificial globe upon which the projection is based, we find the length of one degree of latitude to be $g = \frac{2\pi R}{360}$. We further mark the degrees of longitude on the line AB, accepting that the length of arc of a degree of the parallel of latitude ϕ is $= l \cos \phi$. On the globe, however, we find that the arcs of equatorial and meridian degrees are co-equal, viz. $e = g$, because both equator and meridians are great circles. Therefore, if ϕ be the geographical latitude of the mean parallel circle AB, the divisions to be marked on AB are $e = g \cos \phi$. Through the points thus indicated parallel lines are drawn to AB and to CD, thus making a net of squares.

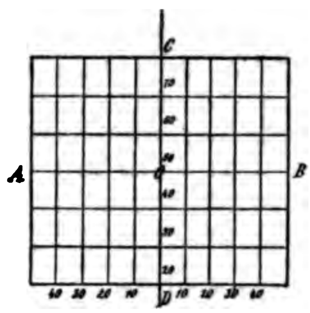


FIG. 39.

Assuming a degree of the meridian to be expressed by 1 cm. (unit of length), the linear expansion of the longitudinal degrees for the corresponding latitudes is as follows ($1° = 1 \cos \phi$ cm.) :—

$\phi = 10°$, $1° = 0{\cdot}985$ cm.	$\phi = 50°$, $1° = 0{\cdot}643$ cm.
$\phi = 20°$, $1° = 0{\cdot}940$ „	$\phi = 60°$, $1° = 0{\cdot}500$ „
$\phi = 30°$, $1° = 0{\cdot}866$ „	$\phi = 70°$, $1° = 0{\cdot}342$ „
$\phi = 40°$, $1° = 0{\cdot}766$ „	$\phi = 80°$, $1° = 0{\cdot}174$ „

This subsidiary net, corresponding in every respect with the astronomico-geographical net of meridians and parallel circles, can be imagined in any possible position with regard to the normal or geometrical network. The two points at which the diameter of the supplementary net cut the plane of the earth are called the *principal points*, corresponding to the poles in the normal condition. The circles corresponding to the meridians are called *principal circles*, and the circles corresponding to the parallels are called *horizontal circles*.

Ptolemy evidently built his geographical work upon the theories of his predecessors. His *Geography*, consisting of eight books, in which he developed his theories of mathematical geography and of cartography, was provided with many illustrative drawings, but whether it ever included maps is doubtful. At any rate, only the drawings have come down to us, reconstructed (partly defectively) upon Ptolemy's system by a certain Agathodemon in the fifth century of our era.

In the eighth book of his *Geography* Ptolemy gave directions how to project the plane of the earth's surface in twenty-six maps. He founded his demonstration upon the projection system of Marinus, which he evidently considered quite exact enough for the purpose, but he allowed that for the representation of large areas this quadrilateral system could not be used. For when the right proportion is preserved for the central parallel of the map, this right proportion cannot exist for the extreme parallels under projection; hence, the greater the latitude of the area to be represented, the greater must be the distortion towards the edges. For instance, if a map represents an area of from 20° to 60° latitude, the mean (or central) parallel will be of 40°, and all degrees of the parallel stand in relation to the degrees of meridian throughout, in proportion of 0·766 : 1. But the true proportion for 20° (lower limit north latitude) would be 0·940 : 1, and for 60° (upper limit), 0·500 : 1. To diminish these defects as far as possible, Ptolemy proposed that for the projecting of large areas conical projection should be applied.

3. *Conical Projection*

A cone is also a suitable body from which to develop the delineations of a sphere upon the plane surface. We strike a conical plane, tangential to the portion of the globe to be represented, and project the different points of the globe's surface upon the tangents. We then develop the cone upon the flat surface. The position of the conical plane should be so that it intersects the earth in the mean parallel AGB (Fig. 40) of the area under projection. The vertex (zenith) C of the conical plane lies in

the extension of the earth's axis $P'P$. The extended meridian planes intersect the axis of the earth, and cut the conical surface into planes at the points marking the extension of their meridians. Thus the extension of the meridian plane $PMGNP'$ cuts the conical surface at points Cm, Gn. If we draw rays of vision from the centre of the globe towards an infinite number of points on a given parallel circle DF, the system of rays produces a second conical surface, and the two intersect each other on a line af, because the vertices of both conical surfaces lie in the respective extension of their common axis.

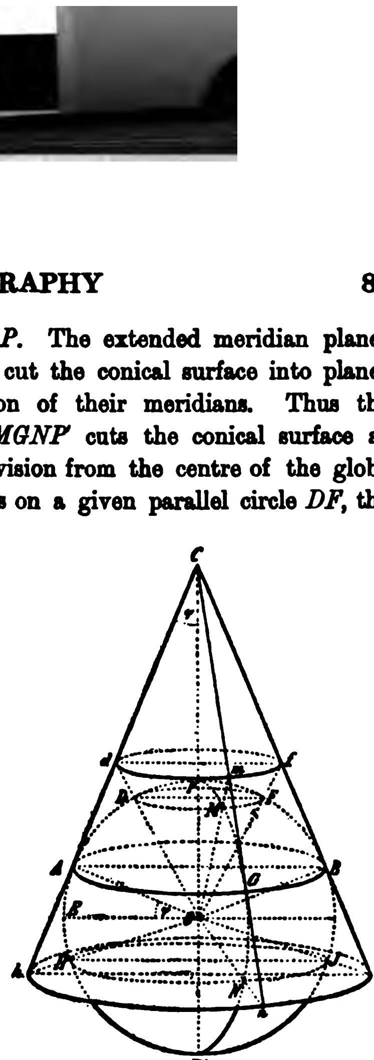


FIG. 40.

Developing the cone, we find that the projections of the meridians appear as converging straight lines, and the projections of the parallel circles as concentric arcs, with a common centre in the vertex of the cone. The parallel circle which formed the circle of contact between the conical and the globular plane is represented in its natural dimensions. This projection is called a *true conical projection*. One of its best-known variants is the *equidistant conical projection*, in which the linear system is preserved, the degrees of latitude are equalised, and the central parallel circle is represented in its true dimensions. To do this we first determine the side CA of the conical surface. Let ϕ (Fig. 40) be the latitude of the mean parallel circle AB, R the semi-diameter of the globe (AO); then is $\angle EOA = \phi = \angle ACO$. Therefore, ACO being a right-angled triangle, $AC = R \cot \phi$. In Fig. 41 we strike a mean meridian CX, and from C as centre, and the semi-diameter $Ca = R \cot \phi$, we describe the arc dd, which

[redacted]

represents the projection of the mean parallel. The length of the degrees of meridian is $1 = \frac{2R\phi}{360}$. We transfer these on to the line CX on either side of a, upwards and downwards. Through the points of intersection we describe concentric arcs, always with C as centre. To construct the meridians for this projection it is best to measure the central angle a,

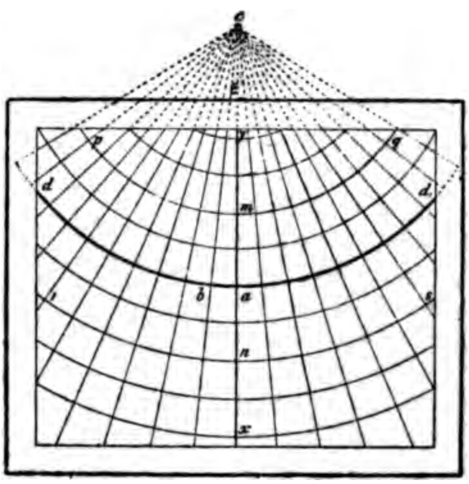


FIG. 41.

which corresponds in size with the degree ab of the mean parallel circle. The length of arc $1'$ of a parallel degree in latitude ϕ is

$$1^\circ = 1 \cos \phi = \frac{2\pi R}{360} = \cos \phi = ab.$$

Since $ab : 2ac \,.\, \pi = a : 360$, and $ab = \cos \phi$, $ac = R \cot \phi$, it follows that:

$$1 \cos \phi : 2R \cot \phi\pi = a : 360, \quad i.e.\ a = \frac{1 \cos \phi \,.\, 360}{2\pi R \cot \phi},$$

and since $1 = \frac{2\pi R}{360}$, $a = \frac{2\pi R}{360} \cdot \frac{\cos \phi \,.\, 360}{2\pi R \cot \phi} = \frac{\cos \phi}{\cot \phi} = \sin \phi.$

If the latitude of the mean parallel be 50°, then sin 50° = 0,766, therefore $a = 0{\cdot}766°$. Repeatedly marking this angle on the line *ca*, towards the top and towards the bottom, the meridians can be drawn without any difficulty on the map under projection.

Equidistant conical projection is to be recommended for countries covering not too extensive an area from north to south, because the projection is simple, and the distortion moderate.

Ptolemy adopted this operation for the projection of the earth from the equator to the northern boundary line of the then known world, and as his mean parallel he again chose the parallel of Rhodes. To minimise the distortions at the upper and lower edges, he introduced certain modifications. He drew the mean meridian as a straight line, and divided it in equal portions to represent the degrees of latitude. The expounders of his *Geography* do not quite agree as to the method employed by him for defining the centre of the parallel on the mean meridian, but according to Delambre this centre was 181° 50′ distant from the equator. Starting from this centre, he described concentric curves through the degrees of latitude, and thus obtained the parallels. But instead of dividing one parallel circle in its proportions as on the globe, he divided four parallels in this manner: the parallels of Thule and of Meroë (on the Nile), as being the extreme parallels of the then known world, and the parallels of Syene (now Assouan) and of Rhodes. He equalised the equatorial and meridian degrees as on the globe, and through every five equivalent points of intersection he described lines of contact, curves, not arcs, and this operation gave him his meridians.

Perspective Projections

General Remarks

There are three azimuthal or perspective projections, the orthographic, the stereographic, and the gnomonic projection. From these three an endless variety of methods of projection can be deduced, which it will be

unnecessary to dwell upon. The three projections named will be considered more in detail, not because of their practical value, which is well-nigh nil, but because of their theoretical value, in so far as the laws upon which they are based help us to understand many doubtful projections, and because some of the properties of projections can be explained far more easily by the laws of perspective than on purely analytical grounds.

In projecting a figure from the spherical on to a plane surface, one would naturally proceed in the same manner as when sketching some

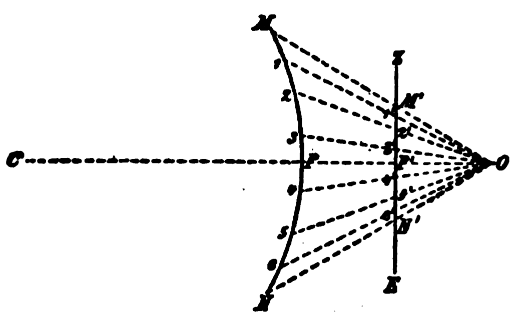


FIG. 42.

extensive view, say a landscape, by applying the ordinary rules o perspective. We imagine the rays of vision, drawn from the eye t different points on the original, as intersecting the plane of representatio in some given point, and we call the position of this point where eac respective ray cuts the plane of representation its point of projectior The plane of representation is perpendicular to the ray of vision whic passes through the centre of the area under projection. To represent portion *MN* of the earth's surface in perspective projection, we assum the eye to be in *O*, somewhere on the prolongation of the semi-diamete *CP* (Fig. 42), which we imagine drawn through the centre of the sectio under projection. From *O* we draw rays to the different points of repr

[redacted]

sentation M, 1, 2, 3, . . . N, and we cut this system of rays by a plane ZE, vertical to CO. The points of intersection M', $1'$, $2'$, $3'$, . . . N' are the points of projection for M, 1, 2, 3, . . . N.

A different kind of projection can only be obtained by altering the position of the eye (point O) with regard to the surface of the sphere. Distance of point O from point P does not alter the character of the projection, but only its scale. The line PO may be extended indefinitely without affecting the representation, except in matter of size. The farther O is from P, the larger will be the scale of projection, but the projection itself remains the same, because the sections $abcd$, rel. $a'b'c'd'$ (Fig. 43), projected on the two parallel planes E and E' resp. from rays at any distance will always retain the same relative proportions, their triangles in point O being one and the same. Thus we get $a:a' = b:b' = c:c' = . . . = x:x'$; x and x' representing the distances of the planes E and E' from point O. Hence $a:b:c:d = a':b':c':d'$; and this proportion applies to all distances within the range of the two planes.

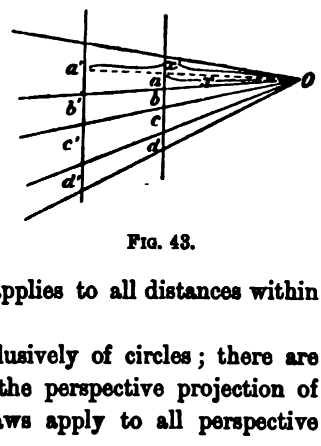


FIG. 43.

On the globe the lineation consists exclusively of circles; there are therefore certain laws to be observed for the perspective projection of these lines upon the flat surface; these laws apply to all perspective projections of the sphere upon the flat surface.

1. All circles on the surface of the sphere, the planes of which pass through the eye, are represented on the map as straight lines.
2. In every perspective projection there must be one meridian projected as a straight line, for a plane can always be drawn through the eye and the earth's axis; and this plane must touch the earth's surface in some meridian, since any plane passing through the earth's axis must be of necessity a meridian plane.

The range of the rays of vision utilised in projecting an object is called the *cone* of projection. The point of vision is the vertex of the cone; the line of intersection which cuts the plane of representation is the projection, the reproduction of the object. Thus a polygon is generally projected as a polygon; a curve, as a curve. If the polygon be rectilineal, the cone of projection is a pyramid. To project a circle KR, the cone of projection is an ordinary (or right) cone. The projection $K'R'$ can only be a circle when the plane of projection ZE is parallel to the plane of the circle KR (Fig. 44).

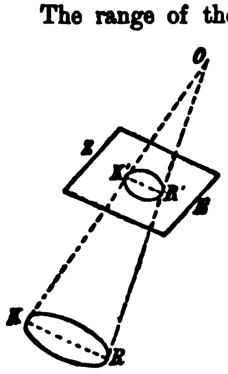


Fig. 44.

Generally speaking, a circle is projected either as an ellipse, a hyperbola, or a parabola. The most usual projection of the circle is the ellipse.

1. *Orthographic or Parallel Projections*

In orthographic projection the eye is imagined at an infinite distance, and the plane of projection is assumed to be perpendicular to the parallel rays of projection. According as these rays are parallel to the axis of the earth, or parallel to the plane of the equator, or parallel to any given semi-diameter of the earth, these projections are distinguished as *orthographic polar*, *orthographic equatorial*, and *orthographic horizontal*, *projections*. If we project the globe on two ground-planes, perpendicular to each other, the one of which (the horizontal plane) is perpendicular to the axis of the earth, then the horizontal projection represents the orthographic polar projection (Fig. 45*a*), and the vertical projection (Fig. 45*b*) represents the orthographic equatorial projection. In the former the centre of the hemisphere under projection lies in one of the poles p (Fig. 45*a*); the parallel circles are projected as circles in their true dimensions, and the meridians as straight lines (semi-diameters). In the latter (Fig. 45*b*) the centre of the hemisphere under projection is on

the equator, for instance, in 0° longitude. The parallels now become straight lines, perpendicular to the axis of the earth, and the meridians appear as ellipses, the earth's axis being their common major axis. The

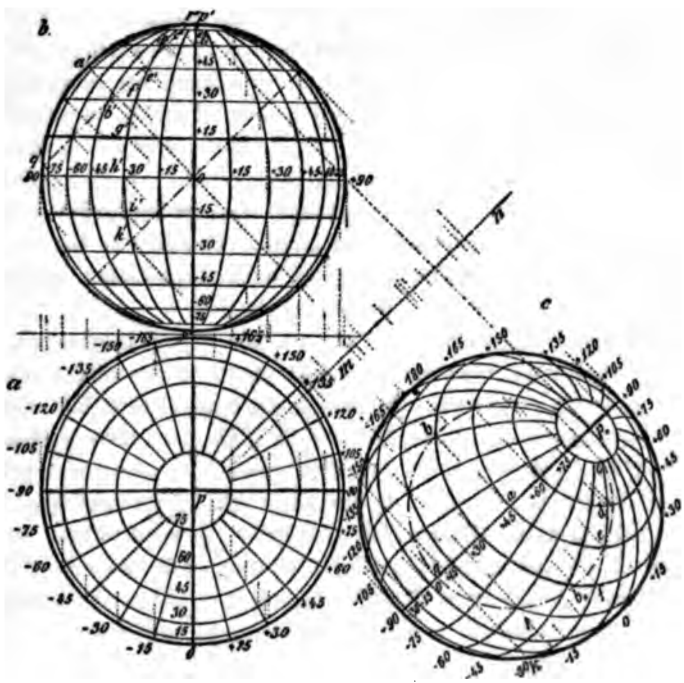


Fig. 45 a, b, c.

central meridian which passes through the point of vision is projected as a straight line coinciding with the projection of the earth's axis. The construction of these ellipses is a very simple operation ; all we have to do is to determine the horizontal projections of the meridian points to correspond with their vertical projections. If the centre of the hemisphere under projection be an arbitrary one, say in A (or its correspondent

point a', Fig. 45*b*), under 45° N. lat. and 90° W. long., we must imagine the plane of projection to be perpendicular to the earth's semi-diameter drawn to this point. This gives us the *orthographic horizontal projection*. This projection can easily be deduced from the orthographic equatorial projection, by assuming that the latter (Fig. 45*b*) is the perpendicular projection of that position of the sphere in which the axis of the earth is parallel to the vertical plane, and forms with the new ground-plane an angle equal to the geographical latitude of A. Such parallel circles are projected as ellipses, and are easily constructed with the assistance of one of the circles parallel to the new horizontal plane, for instance, of circle $q'r'$, or the circle round a, with radius ag. Thus we get for b' the new horizontal projection b respectively 6°. The meridians also become ellipses, with the exception of the central meridian passing through A, which is projected as a straight line through a, parallel to the new ground-plane m, n.

Fig. 45 *a*, *b*, *c*, represents these three orthographic projections. The regions situated in the immediate neighbourhood of the centre of projection are correctly represented in orthographic projection, but since the latitude decreases steadily towards the edge of the map, the representation necessarily becomes contracted and less exact the nearer the point in question is situated to the edge of the map. This projection (Fig. 45*c*) is therefore seldom used. Hipparchus (160-125 B.C.) seems to have been the originator of the orthographic method. It commends itself for the projection of such heavenly bodies (the moon, for instance) as represent themselves to our view of necessity in orthographic projection.

2. *Stereographic Projections*

In stereographic projection the eye is situated at a point somewhere on the globe's surface, and the plane of projection generally passes through the centre of the earth. Hipparchus again invented this method of projection, and Ptolemy gave it the name of *planisphere*.

According to whether the eye be situated at the pole, on the equator, or at any other point, the projection is called *stereographic polar*, *stereo-*

graphic equatorial, or *stereographic horizontal*. The plane of projection passes through the centre of the earth, and is always perpendicular to the central ray of vision—namely, the radius which is drawn to the centre of the earth. In stereographic polar projection, therefore, the plane of projection is situated in the plane of the equator; in stereographic equatorial projection in the plane of the meridian situated at a distance of 90° from the meridian of the point of vision; in stereographic horizontal projection the plane of projection coincides with the plane of the sensible horizon of

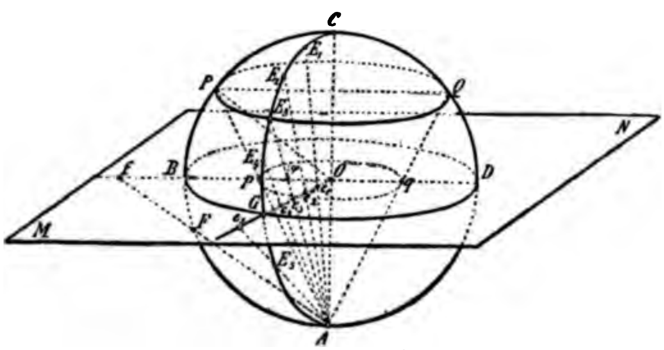


Fig. 46.

the point of vision. The plane of projection can also be moved parallel to these positions, and brought into contact with the earth's surface at some other point.

Stereographic projection necessarily represents that portion of the globe, the centre of which is the point diametrically opposite to the point of vision.

The ancients knew only stereographic polar projection; the Arabs introduced horizontal projection.

Let *ABCD* (Fig. 46) be the globe, *A* the eye, and *MN* the plane of projection. We find that *f* is the stereographic projection of point *F*, because a ray has been drawn from *A* to point *F* on the surface of

the globe, and this ray cuts the plane of projection at f. In the same way q is the stereographic projection of Q. Now, if A be at the same time one of the poles of the earth, the projection is *stereographic polar*. In this projection all meridians appear as straight lines, radiating from point O of the projection. For if we draw the visual rays $AE^1, AE^2, AE^3, AE^4, \ldots$ to points $E^1, E^2, E^3, E^4, \ldots$ of the meridian $CE^1E^2E^3 \ldots A$, the radiation thus obtained forms a plane, coinciding with the meridian plane CGA. This plane can only cut the plane of projection in a straight line OG. The projection of the pole of the earth is in O, and as all meridians must pass through C, their projections must pass through O. Thus OG is the stereographic projection of the meridian CGA, OB of the meridian CBA, etc. The meridian planes CBA and CGA enclose the spherical angle BCG, which is determined by the equatorial arc BG. But BG also represents the size of the central angle BOG, *i.e.* of the angle limited by the projections of the relative meridians. We see then that the rectilinear projections produce angles equal to their degrees of longitude. The equator lies in the plane of projection, and therefore appears in its true dimensions. The parallels appear as circles concentric with the equator, for the visual rays drawn from A to the parallel PQ represent a vertical circular cone intersected by the plane of projection, which is parallel to the base plane, along a circular line pq; and since the radius AC cuts through the centre of all parallel circles, the projections of all the centres of parallel circles fall in O. To determine the semi-diameter r of the projection of a parallel in latitude ϕ, we have to consider that, since $\angle COP = 90° - \phi$, $\angle CAP = \frac{1}{2}(90° - \phi)$. But $\triangle AOp$ shows that if $AO = R$, $r = pO = R \tan\left(45° - \frac{\phi}{2}\right)$.

We now proceed to design a net in stereographic polar projection.

Let $ABCD$ (Fig. 47) be the circle representing the equator, its semi-diameter being of arbitrary length, to suit the size of our paper. We divide the circumference in lengths of from 10° to 10°, and through these points we draw semi-diameters, which represent the meridians. We then proceed to connect the point B of the equator with points $a, b, c, d,$

etc., corresponding to longitudes 10°, 20°, 30°, 40°, etc. Where the lines of contact cut the meridian AC, we get the divisions corresponding to latitudes 10°, 20°, 30°, 40°, etc. Drawing concentric circles through the points thus fixed, we obtain the parallels corresponding to latitudes 10°, 20°, 30°, 40°, etc. For instance, we see that Om is the semi-diameter of the parallel circle of latitude $\phi = 10°$, by comparing the

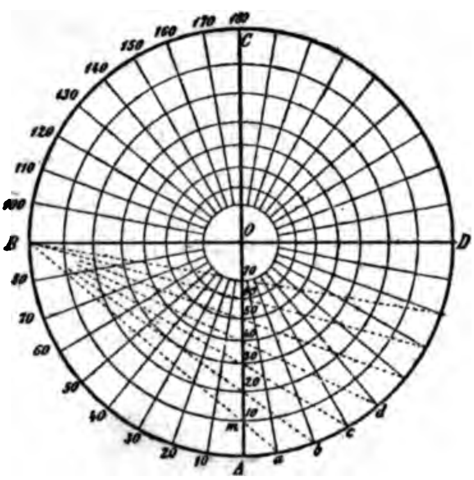


FIG. 47.

triangle OBm. For since $\angle DOa = 90° - \phi$, $\angle DBa = 45° - \frac{1}{2}\phi$ and $Om = OB \,.\, \tan\left(45° - \frac{\phi}{2}\right)$ or $Om = R \,.\, \tan\left(45° - \frac{\phi}{2}\right)$ as above, therefore $Om = r$.

Fig. 47 shows that if C be the north pole, the entire northern hemisphere falls inside the equator, and the entire southern hemisphere outside of it. The projection f of the point F is already at a great distance from the centre of the map, and this distance becomes infinitely great for points situated in the immediate vicinity of A. It is therefore impossible

[redacted]

to represent the entire surface of the earth on one sheet of paper stereographic projection, and the distance of the parallels of the furtl hemisphere from the centre of the sheet of paper is greater in p portion to their geographical latitude.

Stereographic projection has two important properties :—

1. The stereographic projections of all spherical circles not passi through the eye are again circles.

2. All angles on the spherical surface are equal to the angles in t projection, that is to say, the projection is equi-angular.

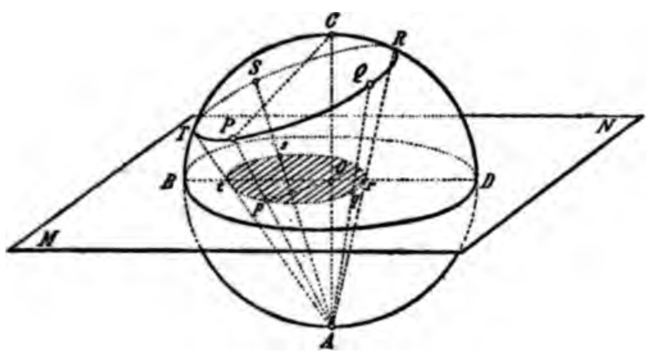


Fig. 48.

To prove the first theorem, we will take Fig. 48. Let $TPQRS$ be spherical circle; MN is the plane of projection, A the point of vision, its opposite point, p the stereographic projection of P. We draw a li joining CP, and another joining Op; we then find that $\angle CPA = COp = 9$ i.e. $\triangle CPA$ similar to $ApOA$; consequently $Ap : AO = AC : AP$, that to say, $Ap . AP = AO . AC$.

We can prove in the same way that

$$Aq . AQ = AO . AC, \quad Ar . AR = AO . AC, \; As . AS = AO . AC;$$

hence

$Aq . AP = Aq . AQ = Ar . AR = As . AS = \;\; . \; . \; . \; . = AO . AC$, i.e. a co stant quantity.

The spherical circle and its projection lie on one and the same spherical surface; but the projection lies also on the plane of projection MN; therefore the projection lies on a sphere, and at the same time on a plane, and consequently must always be a circle.

To prove the theorem of equality of angles, we take Fig. 49. Let xy and uz be circles, and A their point of intersection. We draw tangents AF and AG. The angle FAG is equal to the spherical angle of xy and uz; and FG is the line of intersection between the plane supposed to pass through FAG and the plane of projection. The projection of point A is a. Assuming a to be connected by a line to F and to G,

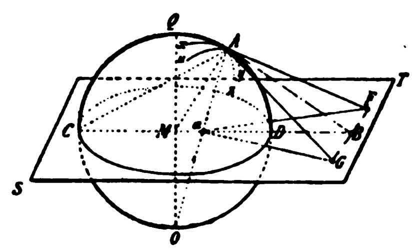


FIG. 49.

aF and aG are the projections of AF and AG, and therefore equal to the tangents of the circles of projection; hence FaG is the projection of FAG.

To prove that $\angle FaG = \angle FAG$ we draw a great circle through A, O, and Q, and draw through A the tangent AB, which cuts FG in B. Since the planes AFG and ST are both perpendicular to the plane $ODAQ$, their line of intersection FG must necessarily also be perpendicular to the plane $ODAQ$, that is, FG is perpendicular to BA, and FG is perpendicular to Ba; hence the triangles ABF, aBF, ABG, and aBG are all right-angled at B. Marking $\angle ACM$ as η, we find that $\angle AMD = 2\eta$ and $\angle AMO = 90^\circ + 2\eta$. Now $\angle aAB$ (as angle between tangent AB and chord Aa) $= \frac{1}{2}\angle AMO = 45^\circ + y$. But since

$$\angle CAO = \tfrac{1}{2}\angle CMO = 45^\circ,$$
$$\angle AaB = CAa + \angle ACa = 45^\circ + \eta,$$

that is to say $\angle aAB = \angle AaB$, and also $aB = AB$,

hence $\triangle aBG = \triangle ABG$

and $\triangle aBF = \triangle ABF$;

therefore $\angle GaB = \angle GAB$,

$$\angle FaB = \angle FAB,$$

and $\angle GaB + \angle FaB = \angle GAB + \angle FAB$,

or $\angle FaG = \angle FAG$.

Stereographic equatorial projection of the earth's surface rests on th same principles. It is easy to see that in this projection (Fig. 50), sinc the eye is at the point A on the equator, all circles (with the exception c the equator $AFQLG$ and the central meridian BQC, whose planes cu through the point of vision A) become circular arcs, and the question i how do we determine their centres and diameters? We begin b delineating the extreme meridian $BGCF$, situated in the plane c projection MN. We draw in this meridian a straight line B, cut at rigl angles by another straight line FG (the diameters); BC represents th projection of the mean meridian BQC, and FG represents the projectio of the equator; the points B and C represent the poles. Supposing w want to represent a system of parallel circles, for instance, at distanc of 15° apart, these divisions will cut the extreme meridian $BJGCF$ on the globe in twenty-four equal parts, thus fixing two points fo every parallel we want to reproduce. A third point on any of the parallels is fixed by determining the projection d of point D on tl parallel circle situated on the mean meridian; this is done by drawing tl ray AD. Since all parallels cut the mean meridian at right angle their projections must also be cut at right angles by BC, *i.e.* the pr jections of all centres of parallel circles lie on the polar axis BC or its pr longation. Now since the extreme meridian must also be a perpendicul the radius OJ (Fig. 51) is tangent to the parallel arc HJ, *i.e.* i respective central point is point P, the point where the perpendicul OJ from J intersects the perpendicular BC. All projections of tl different meridians must clearly pass through the points B and C.

further point is obtained by determining the projection l of the meridian point L situated on the equator (Fig. 50). Since all meridians cut the equator at right angles, it is evident that in the projection all meridians cut the diameter FG at right angles, or in other words, the

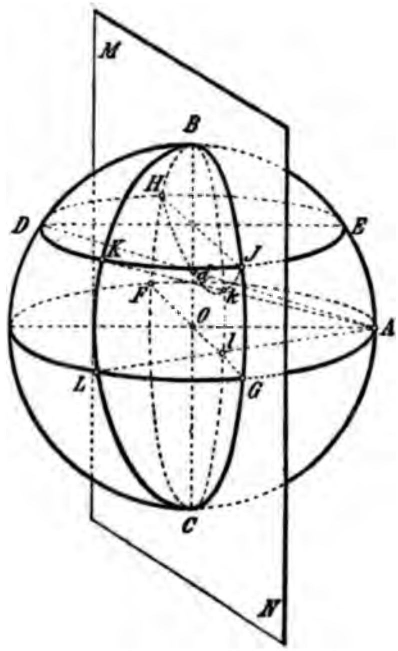


Fig. 50.

several centres of the circles must all lie on the straight line FG or its extension, *i.e.* on the axis of the equator. For instance, to find the centre of the meridian arc which makes a spherical angle of 15° with the extreme meridian, we have only to remember that the projection of this meridian must likewise make an angle of 15° with the projection of the extreme meridian $BFCG$, and the semi-diameter must also describe

an angle of 15° at the point of intersection B. It therefore suffices t produce $\angle OBQ = 15°$; the arc QB drawn from Q gives the projection o its respective meridian. Since $\angle COR = 30°$, $\angle CBR = 15°$, or in oth words, Q is merely the point of intersection of BR and FG.

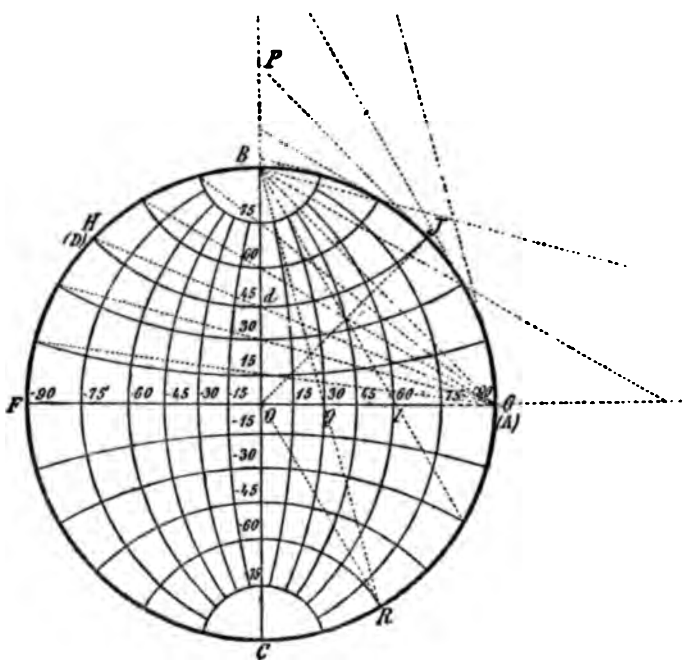


Fig. 51.

To draw the network for stereographic equatorial projection, we descr a circle, its semi-diameter being in suitable proportion to the size of t paper. We draw the polar and equatorial axes in their relative p pendicular positions, and divide the circumference of the extreme merid into the required divisions. The centres of latitude are found

explained in Fig. 50,[1] or this may be done upon the following principle. Let the geographical latitude of the parallel circle $HDJ = \phi$; it follows

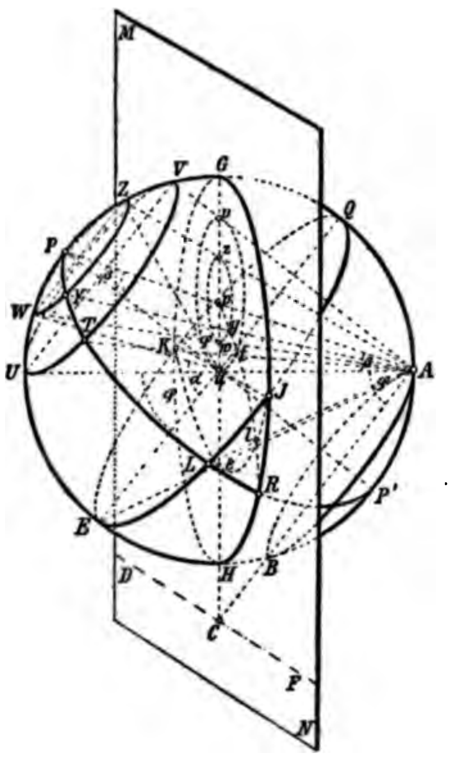


FIG. 52.

that $\angle JOG = \phi$, and also $\angle JPO = \phi$; that is to say, the semi-diameter of the parallel under consideration is $PJ = OJ \cot \phi = R \cot \phi$. If, moreover, $\angle OBQ = \lambda$, *i.e.* if the meridian under projection forms the

[1] The bracketed letters in Fig. 51 refer to the positions of the respective points on the plane of the mean meridian in Fig. 50.

angle λ with the extreme meridian, it follows, since BOQ is a right-angle triangle, that the length of QB, the semi-diameter of the projection of th meridian under consideration, is $QB = \frac{BO}{\cos\lambda} = \frac{R}{\cos\lambda}$, or what is the sam thing, $QB = R \sec\lambda$; and also that $OQ = BO \cdot \tan\lambda = R \cdot \tan\lambda$.

To obtain the *stereographic horizontal projection* of a place, its geographic: longitude being $= 0°$, and its north geographical latitude $= 48°$, we imagi

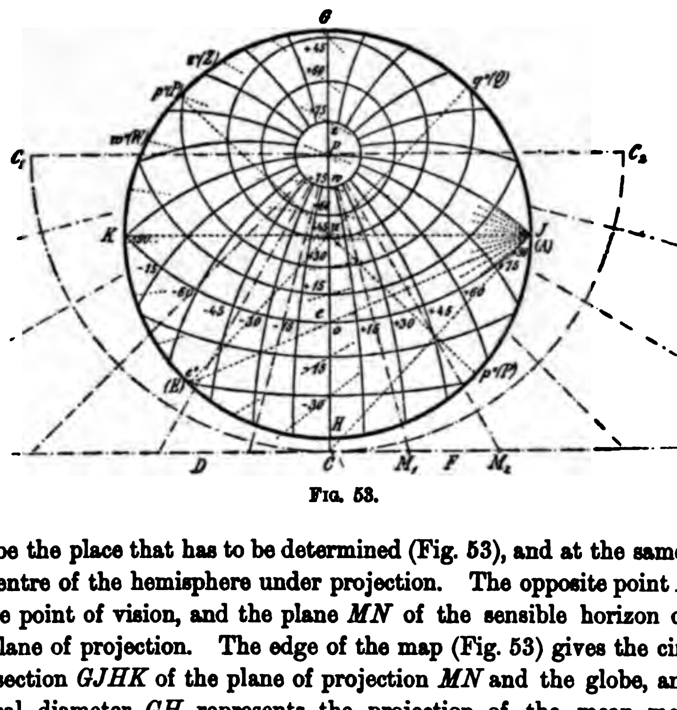


FIG. 53.

u to be the place that has to be determined (Fig. 53), and at the same ti the centre of the hemisphere under projection. The opposite point *A* w be the point of vision, and the plane *MN* of the sensible horizon of *u* the plane of projection. The edge of the map (Fig. 53) gives the circle intersection *GJHK* of the plane of projection *MN* and the globe, and t vertical diameter *GH* represents the projection of the mean meridi *GVZPWUEH* in Fig. 52. Since the plane of the equator *EQ* cuts t plane of projection *MN* by the diameter *KJ* perpendicular upon the li *GH*, the projection of the equator as given in Fig. 53 must pass throu

[redacted]

the extreme points K and J of the diameter perpendicular to GH. The projections of the points of the different parallels situated on the mean meridian $GVZPWUEH$ (Fig. 52) must of necessity fall on GH, and are determined by means of the rays AV, AZ, AP, AW, AU, etc. We notice that in this operation the projection u of U lies in the centre of the boundary circle. Now in Fig. 53 let $\angle Kue'' = 48°$ (geogr. lat. of U); draw a line $p'up''$ perpendicular to $e''uq''$ in u; divide the periphery of the circle of circumference into twenty-four equal parts, beginning at e'', and from the points of division draw rays to J. The points where these rays cut GH will be the stereographic projections of the points on the mean meridian of a system of parallel circles at equal distances of 15°. For instance, if the arc $p'w'' = p'z'' = 15°$, the points of intersection of GH with the rays Jw'' and Jz'' give the stereographic projections of W and Z as w and z, through which points the parallel of 75° north latitude must pass. Since all parallels cut the mean meridian at right angles, this must also be the case in the projection; *i.e.* all centres of parallel circles must in the projection lie on the line GH; thus, for instance, the centre of wz is the centre of the parallel in 75° north latitude.

Amongst the parallels there is also one which is always projected as a straight line, viz. the parallel which passes through the point of vision A. The projection DF of this parallel is nothing but the line of intersection DF of the plane of the parallel AB and the plane of projection MN. Of necessity DF must be perpendicular to GH. In Fig. 53 we find the line DF by drawing a parallel to $e''q''$, through J, its point of intersection C being determined by GH, and a perpendicular being drawn in C on GC. Since all meridians cut this special parallel at right angles, this must also be the case in the projection, viz. all meridians, with the exception of the mean meridian, become circles, cutting DF (Fig. 53) at right angles; hence the centres of these circles must all lie in DF. The meridian perpendicular to the mean meridian is clearly the circle which passes through K, p, and J, its centre lying towards C. To obtain the projection of a meridian which makes a spherical angle of 15° with the meridian just referred to, we must again remember that the

projections of these meridians must likewise intersect each other at 15 and therefore that the rays directed towards the point of intersectio must contain an angle of 15°. Hence if $\angle CpM_1 = 15°$, M_1 must be th centre, and since the projections of all meridians must pass through p, th arc M_1p described from M_1 represents the projection of the meridia under consideration. In order to facilitate the construction of th different angles to pC, we describe a semicircle centre p with pC a radius, draw a perpendicular on Cp, and divide this semicircle, beginnin at C, into twelve equal parts, six on either side of C. This being don the stereographic horizontal projection is easily completed.[1]

By a similar operation to that used in equatorial projection (p. 94), w can prove that the length of the ray of a meridian circle $= \frac{R}{\cos\phi \cos}$ ϕ being the geographical latitude of U, and η the angle contained by th required meridian and the meridian KpJ (Fig. 53). The length of th radius of the parallel circle we express by $\frac{R}{2}\left(\cot\frac{a+\phi}{2} - \tan\frac{a-\phi}{2}\right.$ a being the geographical latitude of the parallel under consideration, an ϕ again the geographical latitude of U.

Stereographic methods of projecting are useful when large portions the earth have to be represented. The hemispheres in our atlases a generally given in this projection (eastern and western, northern an southern hemispheres), also hemispheres of the largest bulk of land water. Sir J. Herschel has pointed out that London is very nearly the centre of the hemisphere of greatest land. But this representati has one great defect. If we look at the network represented Figs. 51, 52, and 53, we see that in some parts of the map t degrees are lengthened out, and that in other places they are co tracted. In stereographic polar projection, for instance, the degrees latitude become narrower as they near the pole; in equatorial projectio on the other hand, the degrees of longitude in the centre of the map a

[1] The bracketed letters in Fig. 53 refer to the corresponding positions of the respect points on the plane of the mean meridian in Fig. 52.

far too small. The principal defect of stereographic projection lies in the exaggeration of the scale of projection between the centre and the edges of the map—a defect which becomes particularly noticeable when, as is often the case in stereographic projection, the representation extends over the surface of an entire hemisphere.

The defect is partly rectified by parallel alteration of the plane of projection, and partly by placing the eye at a certain distance from the surface of the globe. In the latter case we get the so-called external projection. Fig. 54 shows the position of the eye and of the plane of projection for these combinations, in *a* for a stereographic projection in which the plane of projection touches the surface of the globe, in *b* for the external projection in which the plane of projection either passes through the centre of the globe, or in some other way touches it. External projection permits five-sixths of the surface of the earth to be brought under representation.

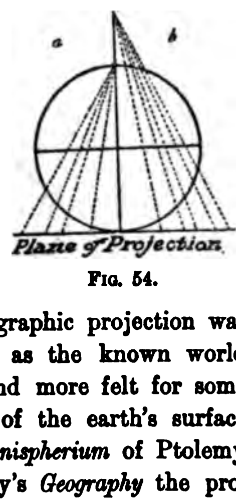


Fig. 54.

The ancients used stereographic projection only for maps of the celestial sphere; it was not until after the discovery of America that stereographic projection was utilised for the drawing of terrestrial maps, for as the known world grew in dimension, the necessity became more and more felt for some method of projection by which larger portions of the earth's surface could be represented. Then it was that the *planispherium* of Ptolemy was thought of, and in a new edition of Ptolemy's *Geography* the projection was first applied to this use. But it was Johannes Werner, a Nuremberg mathematician, who gave it its true vital power (1468-1528).

3. *Central or Gnomonic Projection*

There are traces of the central projectiou as far back as Thales. At least it is not improbable that the representation of the ecliptic in the form of the old sun-dials (gnomon) would lead to the representation of the starry heavens by the same projection. It is doubtful, however, whether the

ancients ever used the method of central projection for drawing
Only during the last century it has come to its full value for the pr

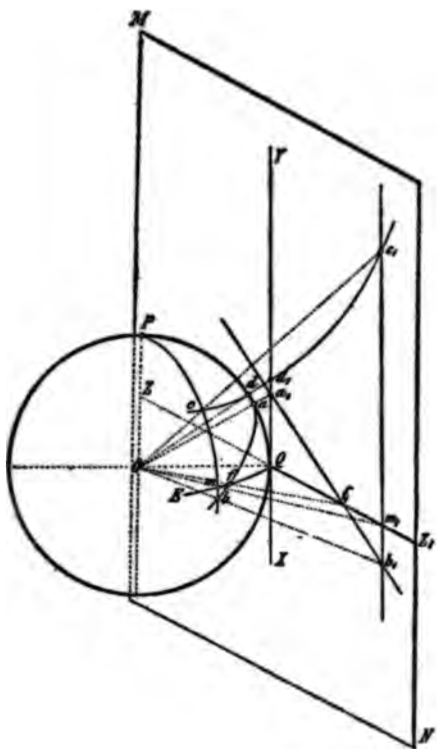


FIG. 55.

ing of maps, thanks to its special property of reproducing the great c
of the sphere as straight lines.

As the name indicates, we imagine the eye to be in the centre o
earth, and the plane of projection to touch the surface of the sp
Central projection can be polar, equatorial, or horizontal, accordi

the plane of projection touches the pole, a point on the equator, or some other point on the surface of the earth.

Let in Fig. 55 EPQ be the earth's sphere, EQ a part of the equator, P the north pole. We imagine the eye to be in the centre O, the plane of projection MN to touch the globe in the point Q of the equator. In order to show that the great circles on the globe are reproduced as straight lines, we have only to consider that the rays directed to the different points of one and the same great circle are also the semi-diameters of that circle, and therefore must lie in the plane of this same circle which cuts the plane of projection in a straight line. The intersection of these two planes gives the projection of the required great circle, and therefore the projection must be a straight line. Hence the equator and the plane of meridian perpendicular to the plane of projection are represented by two straight lines perpendicular to each other. Let XY and ZZ_1 be these two straight lines in the plane of projection MN. The line XY will run parallel to the axis of the globe and touch the globe in Q. This determines the position of the line ZZ_1, for this line must be vertical to XY, and likewise touch the globe in Q.

To form a clear idea of the projection of any other meridian, we must remember that these projections must all pass through the projection of the pole; and as the ray directed to the pole runs parallel to the plane of projection, it can only meet the pole at an infinite distance, and consequently the meridian projections can only meet each other at infinity. Hence it results that meridian projections are represented by parallel lines perpendicular to the equator, and all we have to do is to extend the ray Om directed to the base of any given meridian until it cuts the line ZZ_1 in m. Meeting the line c_1b through m_1, which is perpendicular to Z_1Z, this straight line is the projection of the meridian Pm.

The Parallels

Parallels cannot be drawn quite so easily, because they appear on the plane of projection as conical lines of intersection. For instance, to project the extreme points c and d of the parallel arc cd, the rays

Oc, *Od* must be extended until they cut the corresponding meridian in c_1 and d_1 respectively. The distance of c_1 from the projection of the equator is greater than the distance d_1Q, and this distance is different for every point of the parallel under consideration. Thus we find that not only the degrees of latitude are not equal to one another, but are of different lengths on all meridians, but also that the degrees of longi-

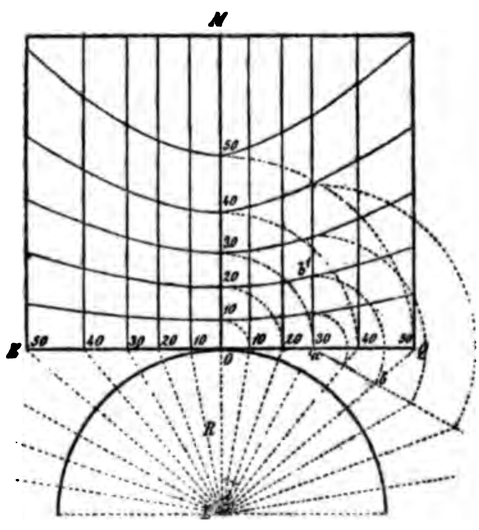


Fig. 56.

tude are not equal to one another. Hence it is not quite so simple a matter to draw the network for gnomonic projection as we have found it for the other methods of projection previously described. But, on the other hand, it is quite easy to project a great circle, of which only two points need be given. If, for instance, we want to get the projection of the great circle passing through *a* and *b*, we lengthen the ray *Oa* to a_1, the ray *Ob* to b_1, and connect a_1 and b_1 by a straight line; this line will represent the required projection.

[redacted]

Let PQ be the prime meridian, Qm the longitude λ of the meridian Pm, hence $\angle QOm = \lambda$, and if R be the radius of the sphere, the projection of this longitude will be

$$Qm_1 = R \tan \lambda.$$

The projection of latitude mc is produced by the right-angled triangle Om_1c_1 if ϕ represents the latitude of c,

$$m_1c_1 = Om_1 \tan m_1Oc_1 = Om_1 \tan \phi.$$

Further, in the triangle QOm_1, $Om_1 = R \sec \lambda$, and hence

$$m_1c_1 = R \sec \lambda \tan \phi.$$

For gnomonic equatorial projection (Fig. 56) we deduce from this the following method of construction. Draw MO perpendicular to EQ (Fig. 56), and with the radius of the globe describe circle centre L, touching the equator in O. Divide this circle, beginning at O, to right and to left in degrees up to 60° or 70°; the intersections of the radii cutting through and extending beyond the points of division are the bases of the corresponding degrees of longitude. In fact, the first of the above equal divisions corresponds to $Oa = R \tan \lambda$.

Now supposing $ab \perp aL$, $\angle aLb = \phi$, and $ab_1 = ab$, then b_1 is the point of intersection of the parallel of latitude ϕ, and the meridian which passes through a, for $ab_1 = ab = La \tan \phi$; but in the triangle $OLaLa = OL \sec \lambda$, hence $ab_1 = R \sec \lambda \tan \phi$, the same as above. Determining several points on one and the same parallel in a similar manner, and connecting them as above, we get the projection of our parallel.

In *gnomonic polar projection* the plane of projection MN touches the surface of the earth in one of the two poles. The planes of meridian produced cut the plane of projection in straight lines passing through P, because the pole is the common point of intersection for all meridians (Fig. 57). The projections of the meridians, however, are also tangents to the globe, hence the angles they contain must be equal. For instance, if Pc be the projection of PCR, Pb that of PBQ, then $\angle bPc$ is the measure

of the spherical angle QPR. A number of rays directed to different points on the parallel AB forms a conical surface, and this is cut in a circle by the plane of projection MN, which is perpendicular to the axis of the conical surface; and since the centres of all parallel circles are situated on the line OP, it follows that the projections of these centres also lie in P. The parallels, therefore, are reproduced as circles with their common centre at the pole. Their radii are determined by the triangle aOP:

$$Pa = R \tan(90° - \phi) = R \cot \phi.$$

To construct the network for this projection (Fig. 57), we draw the

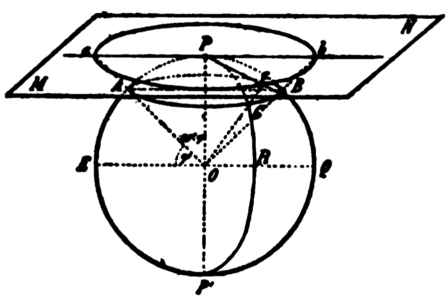


FIG. 57.

meridians as straight lines which intersect in the centre of the map at angles corresponding to their respective longitude differences. From P, the common point of intersection, we draw a length $PO = R$ on any one meridian, and from O we draw straight lines, making with OP angles of 10°, 20°, 30° . . . The points of intersection of these latter with the meridian perpendicular to OP give the points through which the parallels 80°, 70°, 60° . . . must pass. Their centre is in P. For instance, we get for the parallel of $\phi = 50°$ latitude, $Pm = PO \tan POm = R \tan 40° = R \tan(90° - \phi) = R \cot \phi$, the same as above.

We cannot represent an entire hemisphere either by polar or equatorial gnomonic projection all at once on one sheet; for if we look at Fig. 58

we see that the size of the degrees of latitude and longitude increases the farther we go from the centre of our paper. The ray directed to the pole would cut the paper at an infinite distance, and the same is the case with the ray directed to the point on the equator which is distant 90° in longitude from Q. In polar projection the radii of the parallel circles in the lower latitudes become so large that they cannot be represented on an ordinary sheet of paper. Central projection is therefore more suited to the representation of small portions of the globe, and

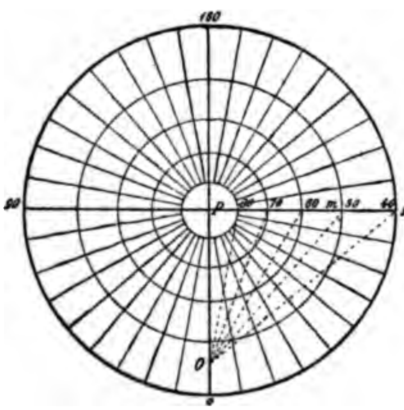


FIG. 58.

in navigation it is a very favourite method. The course of a ship sailing from one point of the globe to another is generally laid down in an oblique direction, or *loxodrome*, *i.e.* a curve which has the property of cutting all meridians at the same angle. The loxodrome is not the shortest line between two points on the earth's surface, but in navigation it is the most convenient, because sailing along this line ensures the mariners remaining within the right course or angle.

The course is the angle made by the direction of the bow of the vessel with the meridian. In long voyages, however, ships try to

follow the shortest line, which is the arc of a great circle connectin
the points of departure and of arrival. This line, called the *orthodrome*, o
"straight" line, is a closed curve or circle. As it always runs parallel t
the curved surface of the earth, it always appears as a straight line wit
regard to the ever-varying horizon, but it does not cut the meridians a
the same angle, because they are not parallel, but converge toward
the poles.

For navigation on a great circle (or orthodrome), gnomonic chart
are used, on which the course of the vessel can be marked as a straigh
line, while in loxodromic navigation Mercator's projection is made us
of (see p. 118).

4. *Loxodromic or Compass Maps*

The maps of the ancients were, after all, at best only distance map
constructed on the purely rectangular co-ordinate system. With th
invention of the compass a great step was gained towards the mor
accurate determination of the direction of places with regard to eac
other, and thus "direction maps" were made, generally known as *compa*
or *loxodromic maps*.

Fig. 59 is a specimen of this kind of map. The system of lines on i
is quite distinct from the graduated network; it is a system of subsidiar
straight lines, consisting of a compass card in the centre of the paper, wit
sixteen principal directions, and a circle of sixteen minor roses, eac
divided into thirty-two parts. The centres of these minor roses lie alon
the periphery of a circle concentric with the central rose, and on th
points where the sixteen principal rays of the central rose touch th
imaginary circle. Each of the thirty-two divisions of the minor roses i
called a point, and has an angular value of $360/32 = 11° 15'$.

In order not to render the maps indistinct by so many lines, thes
used to be drawn in colours—the eight principal directions (N., N.E
E., S.E., S., S.W., W., N.W.) in black; the half-points (N.N.E., E.N.E
E.S.E., etc.) in green; and the quarter points in red. The mile scale, fc

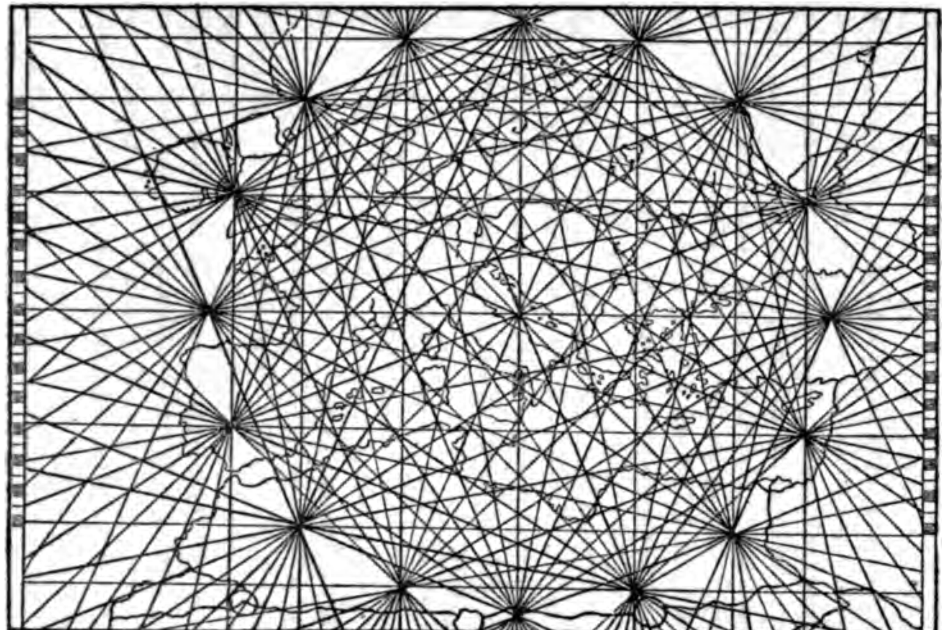


Fig. 59.—Mediæval Mariner's Chart.

measuring distances, was generally drawn along the vertical edge of tl map.

From the earliest times sailors had been accustomed to record tl directions and distances of the different places they visited, and collectio of such notes were called variously *periples*, *stadiasmes*, *portulani*, and *se books*. These sea-books were from time to time corrected, and the materi therein contained must in the end have been fairly correct. In the lar Italian ports along the Mediterranean there were people who ma hydrography and cartography their profession. They collected the se books and based their charts upon them, marking for each journey fro the same starting-point the different directions and distances by means scale and compasses. For instance, if they found for a certain place *B* distance of 50 miles N.E. of another place *A*, they marked *B* on the m 50 miles to N.E. of *A*, etc. When, in determining the position of plac from different starting-points, differences arose as to the exact position the place, they split the difference by taking the mean value of tl different entries.

Amongst the oldest known specimens of these mediæval maps are tl nautical atlas of Genoa, known as the *Atlas Luxoro* (first half of thirteen century), and the so-called Pisan charts (end of twelfth century). Specime of later date are extant in all the principal libraries of Europe, and tl greater part of these cartographical monuments have been reproduced a made accessible to the public. Foremost among the Italian cartographe stands Pietro Visconte of Genoa (1318). Italian charts naturally repı sented only those regions to which their navigation was confined, tl Mediterranean coasts and the Atlantic face of Europe. The coast-lin for these parts were so accurately drawn that Italian charts were used over Europe, and were not superseded by more modern productions un the seventeenth century.

Besides the Mediterranean charts, we have handed down to us fro the Middle Ages a large quantity of world maps (*mappa mundi*). The drawings, though highly interesting as curiosities, are of no practic cartographical use. The later world maps, as for instance that of F

Mauro (1457), are of value, in so far as they show how the Italian sea-maps were converted into land-maps.

During the Middle Ages the great work of Ptolemy was no longer understood, and was at length lost, but the scientific zeal of the learned men of the Renaissance unearthed the ancient treasures of scientific cartographical knowledge, and Ptolemy's *Geography* went through several editions. His system became universally known and adopted in western lands. It was found, however, that the so-called Ptolemaic maps in their traditional form contradicted the teaching of the Ptolemaic text, and hence several attempts were made to reform the theory of cartography or map-drawing. The first who ventured to suggest that a new method of projection should be adopted for the next edition of Ptolemy's maps was a Benedictine monk, by name Dominus Nicolaus, erroneously called Donis.

The network of his pseudo-cylindrical projection consists of straight-lined meridians and parallels, as is the case in plane maps, but with this difference, that not only the mean parallel circle, but also the extreme parallels of the map, are divided in the correct proportion. This imperfect manner of projection, however, found but little favour, because its straight-lined parallel circles did not show up sufficiently the spherical shape of the earth; and once again the conical projection was resorted to, for a new edition of Ptolemy's work (Rome, 1507). But instead of circumscribing the cone to the sphere, they imagined the cone to be inside the globe, assuming its vertex to be at the pole A, and its base-line at the equator BC (Fig. 60). In developing this conical surface the question is to define the angle a. To do this we have the proportion,

$$bc : 2\pi ab = a : 360,$$

only in the present instance $ab = AB = \sqrt{2R^2} = R\sqrt{2}$. Let bc contain an arc of 90° longitude; we find that, because the equator is here to be represented in its natural dimensions, $bc = \frac{\pi R}{2}$, hence

$$\tfrac{1}{2}\pi R : 2R\sqrt{2}\pi = a : 360\text{; and } a = \frac{90^\circ}{\sqrt{2}}.$$

With the semi-diameter $R\sqrt{2}$ we therefore describe the arc *bc*, and al the mean meridian; from the vertex of the sector, we apply on both sid of the mean meridian the angle $\frac{a}{2}$.

The development *bc* of the equatorial segment, the latter being giv in its natural dimensions, can be divided into equal parts, representi degrees of longitude. The pole of the earth is in the centre *a* of t sector *bac*. The degrees of the meridian are made equal to one anoth and from *a* as centre we describe concentric arcs through the poi of intersection to represent the parallels. Extending *ab* and *ac* in t

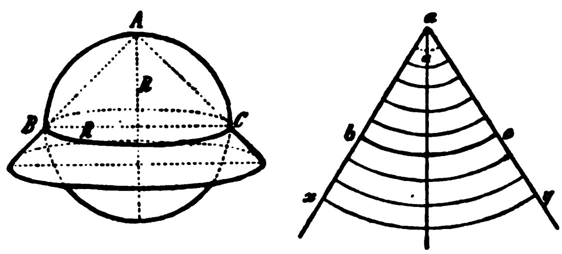


FIG. 60.

direction *ax* and *ay*, the degrees of south latitude from *b* and *c* to *x* a can also be entered. This projection therefore allows of a drawin extend over southern latitudes as well, but it evidently did not find n favour, for there is only one map preserved to us in this projection, dr by Johann Ruysch (1508).

The end of the fifteenth and beginning of the sixteenth century s great change in map-making. The known world had become a great larger, and cartographers devised different methods to represent the e world on one sheet of paper. The heart-shaped projection of Berna Sylvanus, really invented by Johann Stab, but first applied by Sylv is the best known of the methods of projection of this time. The r sentation (Fig. 61) shows the effect produced.

A straight line represents the central meridian, divided at equal distances to represent the degrees of latitude. The centre of the parallels of latitude is placed at 100° from the equator (instead of 181° 8′ of Ptolemy's system), and from there concentric curves are described, cutting through the points of division on the middle meridian. . The degrees on the

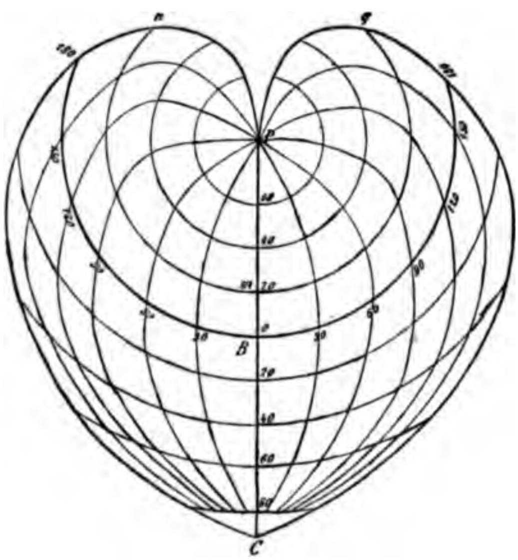


FIG. 61.

equator are then made equal to the degrees of longitude, after which three southern parallels, and eight northern, are subdivided in their true proportions. The lines cutting through the points of division produce curves, which are the projection of the meridians. Thus the map assumes the shape of a heart, with the point at the bottom missing. Several modifications of this projection were proposed by different mathematicians, amongst whom we must mention J. Werner and Peter Bienewitz, better known as Peter Apian (1495-1552). To this same era belong

several other projections, with slight differences of construction, and the double heart-shaped projection of Orontius Finäus, a French mathematician (1531), which was adopted by Mercator with slight modifications. Fig. 62 gives this projection.

From the angular points of an equilateral triangle ABC (Fig. 62) as centres, describe three arcs AC, AB, BC; A representing the pole and BC a quadrant of the equator; AB and AC being two quadrants of meridians at distances of 90° longitude. Draw a straight line AD joining the pole A with the point bisecting the equatorial quadrant BC. This straight line

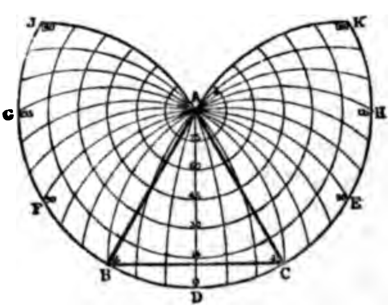


FIG. 62.

represents the middle meridian on the map, and we proceed to divide it into equal parts. Through these points of division describe concentric arcs, with A for their common centre; this gives the circles of latitude. Finally divide the equatorial arc BC and the parallel circular arcs of 45° latitude, concentric with it, into equal parts, and through these points of division and the pole describe arcs to represent the meridians. To enable one to represent an entire hemisphere, this construction has to be enlarged, by extending the equator and all circles of latitude on either side of the middle meridian. With the semi-diameter DA, from D as centre describe arcs AE and AF; this gives the meridians at 90° from the central meridian on map. With the same semi-diameter, from the points B and

C as centres describe arcs AG and AH; and finally from E and F as centres describe arcs AJ and AK, these latter being 180° distance from the central meridian. Continuing in this manner, and with the same semi-diameter, the entire network of meridian circles can be represented.

Finäus, and after him Mercator, projected the entire world on this double heart-shaped network.

The Reformation of Cartography

Mercator, the Reformer of Cartography

Gerhard Kremer, better known as Mercator, was born at Rupelmonde, March 5, 1512. He was educated at the University of Louvain, where he distinguished himself under the celebrated mathematician, Gemma Frisius, in the construction of astronomical instruments, terrestrial globes, and maps. During the religious disturbances in the Netherlands he was banished to Duisburg, where he spent the rest of his life, and died in 1594. Thus, although a Belgian by birth, he belonged to the German nation during the times of his greatest mental activity.

Many of his earlier maps, belonging to the Belgian period, have now been recovered, but they are for the greater part copies of those of the old masters in cartography. The first of his original creations appeared in Duisburg in 1569, being a *Map of Europe*, closely followed by his *Map of the World*, upon which his European fame is based. In the history of cartography, he is the man who closes the old and opens the new era. His time still laboured under the ban of over-estimating the Ptolemaic formal method: Mercator, with his *Codex of Ptolemy's Twenty-seven Maps*, gave the ancient master, once and for all, his true place among the scientists of the world, and he himself stepped forth as a new Ptolemy. All that his time had produced, the entire cartographical movement of this age, was summed up by him in his Atlas, and moulded into a system

upon which all modern cartography is based. In the year 1585 appeare the first part of his *Collection of New Maps of Modern Geography*;[1] but h did not live to see the collection completed. To this day, however, a atlases, in name and in method of construction, bear witness to th principles laid down by him.[2]

Mercator's (Equiangular) Cylinder Projection

We have already (p. 109) alluded to the fact that a ship sailin from one place to another, on a loxodrome, describes a curve which cu all meridians at the same angle. Now to represent the loxodrom line of the ship's course by a straight line, which is the only practic representation for nautical purposes, we must have a chart on whi the meridians are parallel to each other, and on which the angles of t spherical surface are given in their natural dimensions.

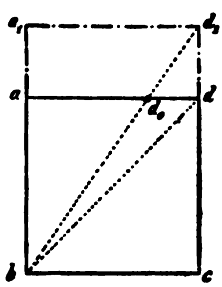


FIG. 63.

In the fifteenth century plane charts were adopted for maritin purposes, but they were not equiangular. T seafaring men of the time heeded this not at fir but gradually they became aware that the cour prescribed on the chart did not lead exactly the intended point of destination. In vain d the best mathematicians attempt to correct the errors of the plane charts; but Mercator solv the riddle. He realised that the mistake lay the construction of the network. Let *abcd* (F 63), in the square plane chart, represent the p jection of a very small segment *ABCD* of globe, contained between two very near meridians and parallels, of 1′ difference of longitude and latitude; and let *m* be the true longit of the equatorial arc, situated between the two meridians *AB* and (

[1] *Atlas sive Cosmographicae ad mentem Ptolemai restitutae et emendatae*, Duisburg, 1

[2] The Atlas contains, besides maps, cosmographical and other dissertations. It completed by Hondius in 1607. Several of the maps had been previously publi separately.

This would read on the plane chart,

$$ad = ab = bc = cd = m = \frac{2\pi R}{360 \,.\, 60}.$$

Supposing the arc AD to have latitude ϕ, this would be represented on the plane chart by $ad = m$, while on the globe it would only have length $m \cos \phi$. By making ad_0 equal to $m \cos \phi$, the angle abd_0 represents the true size of the respective angle on the globe, while on the plane chart this angle is represented by $\angle abd$. To make the map or chart equiangular, and at the same time allow the parallel arc AD to retain the length m of the corresponding equatorial arc, the projection of D on the map must fall at the point of intersection d_1 of the extended lines bd_0 and cd. The triangles a_1bd_1 and abd_0 being similar, it follows that

$$a_1b : ab = a_1d_1 : ad_0 = m : m \cos \phi = 1 : \cos \phi; \text{ that is to say,}$$

$$a_1b = \frac{ab}{\cos \phi} = ab \,.\, \sec \phi = m \sec \phi.$$

Therefore, to ensure conformity in a map with rectilinear meridians and parallels of latitude, perpendicular to each other, and in which a degree of longitude in all latitudes has the same magnitude, *i.e.* its value on the equator, it is necessary that the length of a degree of the longitude in the different latitudes be enlarged in the proportion of the secant of these latitudes. This is what Mercator discovered and carried out in his map of the world (1569), and hence all maps constructed upon this principle are called Mercator maps.

The distance of any two succeeding parallel circles of latitudes $1°, 2°, 3°, 4°, \ldots$ will therefore be $= m \sec 1°, m \sec 2°, m \sec 3°, m \sec 4°, \ldots$ *i.e.* the distance apart of two such parallel circles increases in proportion to the secant of their geographical latitude. The distance x of the parallel of latitude ϕ from the equator is consequently determined by

$$x = m(\sec 1' + \sec 2' + \sec 3' + \sec 4' + \ \ldots \ + \sec \phi').$$

According to the higher analysis, however,

$$\sec 1' + \sec 2' + \sec 3' + \sec 4' + \ . \ . \ . \ + \sec \phi'$$
$$= \frac{180.60}{\pi} \log \tan\left(45^\circ + \frac{\phi}{2}\right);$$

the equatorial distance of the parallel circle in latitude ϕ (m being the length of a minute of arc at the equator) is therefore expressed by

$$x = m\frac{180.60}{\pi} \log \tan\left(45^\circ + \frac{\phi}{2}\right) = R . \log \tan\left(45^\circ + \frac{\phi}{2}\right).$$

The following table is based upon this formula. It shows the distances of the different parallel circles from the equator, expressed in equatorial minutes. These ratios are also called meridional parts or enlarged latitudes.

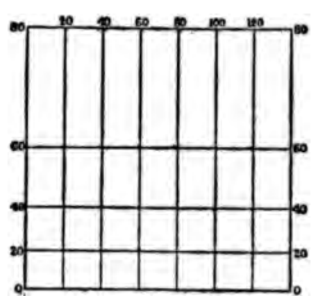


FIG. 64.

Fig. 64 shows the increasing magnitude of degrees of latitude; the squares at the equator become parallelograms of ever-increasing altitude upon the same base. Since for $\phi = 90^\circ$, $\tan(45^\circ + 45^\circ) = \infty$, it is evident that the pole cannot be represented upon a Mercator's map, and indeed this projection is not used for very high latitudes.

Geogr. Latitude.	Parts of Meridian.	Geogr. Latitude.	Parts of Meridian.	Geogr. Latitude.	Parts of Meridian.
1°	60·0	29°	1819·4	56°	4073·9
2	120·0	30	1888·4	57	4182·6
3	180·1	31	1958·0	58	4294·3
4	240·2	32	2028·4	59	4409·2
5	300·4	33	2099·5	60	4527·3
6	360·7	34	2171·5	61	4649·2
7	421·1	35	2244·3	62	4775·0
8	481·6	36	2318·0	63	4904·9
9	542·2	37	2392·6	64	5039·4
10	603·1	38	2468·3	65	5178·8
11	664·1	39	2544·9	66	5323·5
12	725·3	40	2622·7	67	5474·0
13	786·8	41	2701·6	68	5630·8
14	848·5	42	2781·7	69	5794·6
15	910·5	43	2863·1	70	5965·9
16	972·7	44	2945·8	71	6145·7
17	1035·3	45	3029·9	72	6334·8
18	1098·2	46	3115·6	73	6534·4
19	1161·5	47	3202·7	74	6745·8
20	1225·1	48	3291·5	75	6970·3
21	1289·2	49	3382·1	76	7210·1
22	1353·7	50	3474·5	77	7467·2
23	1418·6	51	3568·8	78	7744·6
24	1484·1	52	3665·2	79	8045·7
25	1550·0	53	3763·8	80	8375·2
26	1616·5	54	3864·6		
27	1683·5	55	3968·0	90	∞
28	1751·2				

With the assistance of this table it is quite easy to project the network of a Mercator's map, for by subtracting two successive ratios one gets the magnitude of the relative degrees of latitude, expressed in minutes of longitude. For instance, to draw the network for a sheet extending from 20° to 30° longitude east of Greenwich, and 40° to 50° north latitude, we draw a straight line along the lower edge of the paper, which we divide into ten equal parts, corresponding to the degrees of east longitude 20° to 30°. Upon the points of division we draw the meridians, and divide at least one of the degrees of longitude into sixty parts to enable us to read off the minutes. From the above table we take the parts of the

meridian corresponding to the given degrees of latitude, and find th
difference between them, thus:—

Lat.	Mer. parts according to Table.	Difference.
40°	2622·7	78·9
41	2701·6	80·1
42	2781·7	81·4
43	2863·1	82·7
44	2945·8	

The straight line drawn along the lower edge of the paper represe
the parallel of latitude 40°. We now proceed to enter 78·9 m. of lon
tude ($=1°\ 18·9'$) on the meridian, touching the parallel at 40°; wh
the point of the compasses cuts the meridian we enter the 41st degree
latitude. From the 41st we then reckon 80·1 min. of longitude, and fi
the 42nd, etc.

Mercator's projection of increasing latitudes (*Carte réduite, Ca
esferica*) is chiefly useful in loxodromic navigation. It is the simpl
method to determine the course of a vessel from one part of the globe
another, and to measure the distance to be traversed. To determine
course on the chart, we connect the point of departure and the point
destination by a straight line, which represents the loxodrome to
traversed. On nautical maps, a great many compass-roses are dra
and all we have to do is to draw a straight line through the centre of
nearest one, parallel to the one already drawn. The point of the comp
with which this straight line coincides is the required course to be follo
by the mariner.

To measure the distance between two points we take the map dista
between them with our compasses, and mark them on the scale
latitudes, half upwards and half downwards, starting from the m
latitude between the two places. We then read off the number
minutes of arc contained between the points of the compasses.
number of minutes of arc gives the number of nautical miles, sixty

which go to make one degree. If the difference of latitude between the given points be too great, we divide the distance to be measured on the map into a certain number of parts, and measure each part separately in the above manner.

During the voyage the seaman verifies his geographical position at least once a day, to make sure that he has not deviated from the right course through currents, bad steering, etc.

To mark on the chart the point of arrival of some vessel, a straight line is drawn from the point of departure on the map, which makes with the meridian the given angle of course of the ship, and on this straight line the distance is marked, subtracted from the meridian scale, as estimated by mean latitude, in the direction of the line of course. The extreme point of the distance thus determined is the point of arrival.

Mercator's projection was intended to be of great help to the sailor; nevertheless it was long enough before his charts obtained general acceptance, and as a matter of fact the universal acceptance of this projection, and of the minute of latitude as the nautical mile, is an achievement of the nineteenth century. As a projection for land-map Mercator's theory fared no better, but has only become popular in the course of the last century; it has now, however, become so thoroughly established that in our days it would be difficult to find a world map projected on any other plan. For survey maps of the world, Mercator's projection certainly gives the most correct delineation of countries, and the only danger we run is to forget the fact that the size of regions near the poles is necessarily always much exaggerated; but for these regions another projection, described below, is used.

Mercator devised several other projections, the most important of which is the equidistant projection. On his large sea-map (1569) the polar regions could not be projected, so on a side map he represented the polar regions by the following method. Round the pole *N* as centre he described the circles of latitude at equal distances from each other. The meridians were represented as straight lines drawn through the pole and cutting one another at the same angles as they do on the globe. The Italians

[redacted]

and the projection [illegible] the French call it after their geographer, [illegible] ([illegible]), who first applied it to represent the northern [illegible]. The limit of the projection is [illegible]. For representing

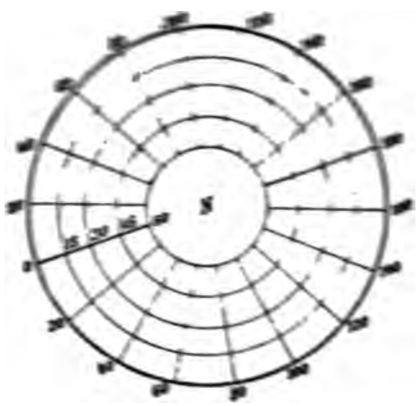


FIG. 65.

large tracts of land at a considerable distance from the centre of the map, the distortion is far too great. Mercator realised this, and never used the projection for maps extending over more than 20° from the pole.

FURTHER MODERN PROJECTIONS

Equi-areal Projection

We saw in the commencement (p. 72) that the arc AB of the terrestrial sphere is reproduced on the artificial globe in the reduced proportion $\frac{r}{R}$. Designating the arc ab of the artificial globe as a, and the arc AB on the terrestrial globe as A, we get

$$a = \frac{r}{R}A.$$

If we suppose a square with the very small arc A, its area will be A^2, and the area of its projection will be a^2. But

$$a^2 = \frac{r^2}{R^2} A^2.$$

Take another square, the one side of which is on the natural globe B, and in the projection b. We get

$$b^2 = \frac{r^2}{R^2} B^2, \text{ consequently } a^2 : b^2 = A^2 : B^2.$$

We see that the areas on the artificial globe are exactly the same in proportion as their originals on the real globe; their proportional size

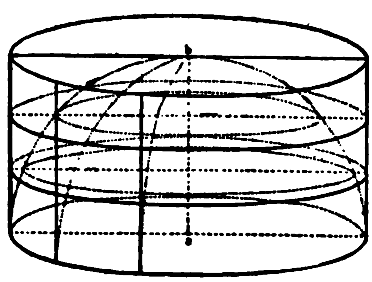


FIG. 66.

remains unaltered. If a method is to be found by which a representation can be made on the plane (flat) surface, in which this property is maintained, we should then have an equi-areal or equivalent representation.

Several efforts were made to obtain this, but the real value of equi-areal projection was not properly understood until J. H. Lambert (1728-1777) made an analytical examination of the laws of map-making. He devised the *equi-areal cylinder projection*, also called *isocylindrical* projection.

In this projection we imagine the earth to be surrounded by a right circular cylinder touching it at the equator (Fig. 66). Instead of directing rays from a certain position of the eye, we imagine the planes of the

meridians to be extended until they touch the cylindrical surface, in equidistant straight lines, parallel to each other. Extending in like manner the planes of the parallels of latitude, these latter will cut the cylindrical surface in circles parallel with and equal to the equator. Now, if we further imagine the cylindrical surface rolled out on the plane, we obtain a network of straight lines, perpendicular to one another, in which the meridians are equidistant, while the distance apart of the parallels of latitude decreases as the sines of their latitudes (Fig. 67), being less the nearer they are to the pole. Thus a network is produced just the reverse of the Mercator net, of increasing latitudes. But like the Mercator projection, it can only be used for representing zones in the immediate neighbourhood of the equator.

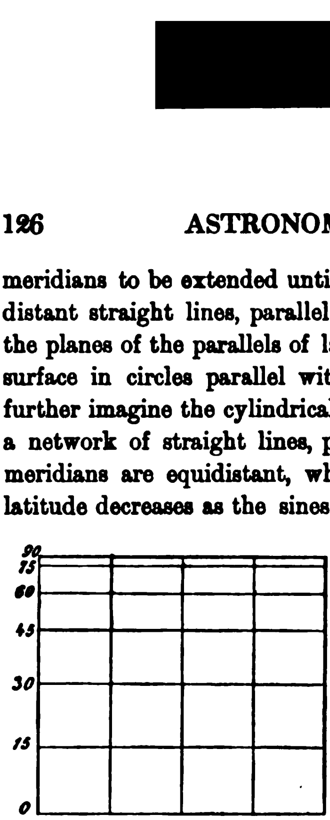


Fig. 67.

This simple projection must necessarily be equi-areal, for the height *ab* from equator to pole of the cylinder circumscribed in Fig. 66 is equal to the semi-diameter of its base, hence equal to the semi-diameter of the globe; its surface area is therefore $=2\pi R^2$. But the surface of the hemisphere is also $=2\pi R^2$; therefore the area of the developed cylinder plane is equal to the area of the hemisphere, and consequently the area of a small segment of the flat surface is equal to the area of a corresponding segment of the hemisphere, *i.e.* the projection is equi-areal.

Another important device of Lambert's was the *equi-areal-azimuthal* projection. In this the plane projection touches the centre of the segment of the globe to be represented, and it is required that all points which on the globe are at equal distances from the point of contact should in the projection also lie in a circle round the centre of the map, and that each point both on the globe and on the map should remain in the same relative position or azimuth, with regard to the point of contact. To make the azimuthal projection equi-areal, the zones on the globe

[redacted]

concentric with the point of contact must be equal in area to the circles in which their projections appear on the map.

Equi-areal azimuthal projection is more suited for the representation of entire continents than the projections now used in most atlases, those of Bonne and Flamsteed. Fig. 68 shows a network in equi-areal azimuthal projection for the mean latitude of 45°, representing the contours of Asia and Europe, the same as shown in Bonne's projection (Fig. 69). The

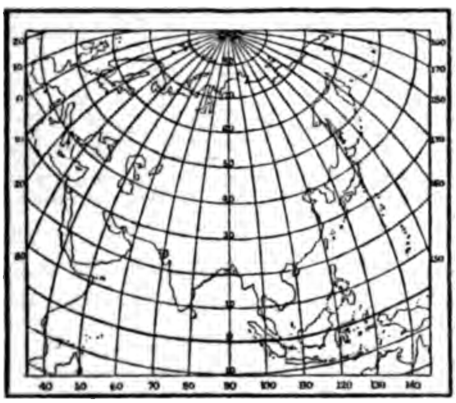


FIG. 68.

shape of Europe in Lambert's projection speaks clearly for the advantages of the latter's method.

The *homalographic* projection (Fig. 70) is chiefly suitable for a representation of the entire world. It is on much the same principle as the Sanson-Flamsteed projection. The central meridian is represented as a straight line of its true length, and the parallels of latitude are straight lines perpendicular to the central meridian, also in their true length. The representation is therefore equi-areal. On the same principle as Sanson's projection, the meridians are described as ellipses, with the projection of the central meridian for their common major axis. To construct the net

the equator is divided into equal parts, and the meridians are ellipses i

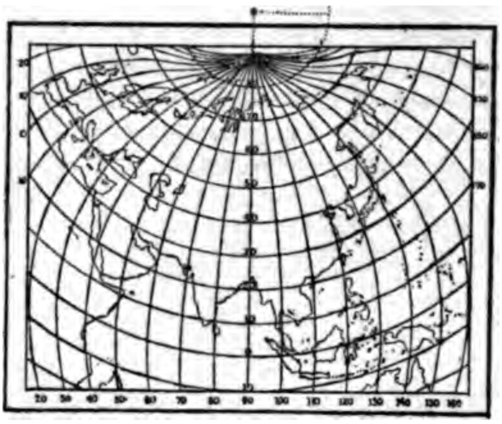


FIG. 69.

secting the poles and the points of division on the equator, but con to Sanson's projection, the central meridian is not divided into

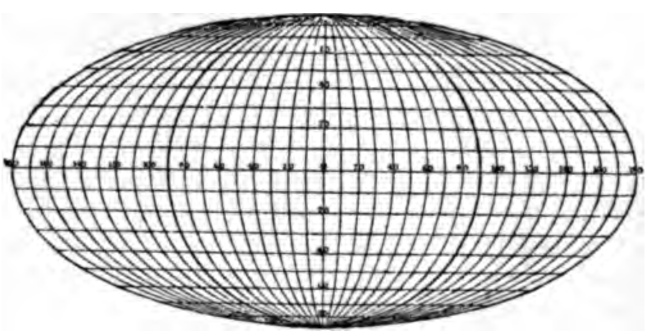


FIG. 70.

parts. The parallels draw closer together as they approach the pol not to the extent of spoiling the distinctness of the representation.

Amongst the later improved projections we further mentio

polyconical projection used for the coast survey of North America. This is a modification of the ordinary conical projection.

Star-shaped maps are another variety of the same projection. The star shape was used by Jäger to represent the north polar region (Fig. 71). Afterwards it was improved upon and simplified by A. Petermann, in that

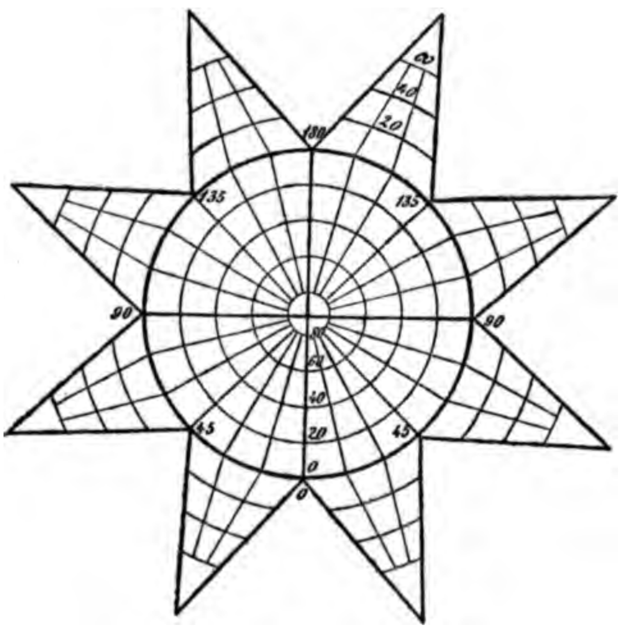


FIG. 71.

he projected the northern hemisphere entirely upon the principle of equidistant projection, and divided the equator into eight equal parts, upon which isosceles triangles were constructed, giving the whole the appearance of a star. The parallels of latitude are described round the north pole as concentric equidistant circles. The rays of the star form the meridians, and the points of division on the equator are joined by chords

to the extreme points of the radii. Other modifications of this projection were given by Berghaus, Steinhauser, etc. They consisted chiefly in a different division of the equator.

On the Choice of Projections with Smallest Distortion

In map-making it is naturally desirable to choose a projection which involves the least possible distortion or alteration either in angles, distances, or surface, and it depends upon the special object of the map which two of these three elements have to be sacrificed to the benefit of the third. We exclude those projections which are merely preferred because of some distinctive property, such as Mercator's cylindrical projection, because of the representation of loxodromes as straight lines; the gnomonic projection, because of its rectilineal great circles, *i.e.* shortest distance between any two given points on the earth; and the orthographic projection, used for representing the lunar surface.

Because of the spherical form of the earth, it is obvious that the angular, distance, and surface distortions can never entirely be made to disappear in any projection on the flat surface; but for high-class maps it is a question of very great importance to decide by which projection the nearest approach to conformity can be obtained in two of its elements, where the third has been brought to the highest degree of equality.

For the representation of an entire hemisphere, all cylindrical and conical projections are necessarily excluded, for in them regions situated near the extreme boundary lines are very much shortened, or else places which on the earth are comparatively close together appear on the projection as at great distances from each other. After dismissing these projections, it remains further to decide whether the greater stress should be laid on the correct representation of angle, distance, or surface.

No projection is equidistant except when referring to distances from the centre of the map. We therefore cannot speak of maintaining distances in general, but only of maintaining distances with regard to the centre of the map. Loppritz calls this *mean distance*.

In order to keep the angles unchanged and to reduce the distortion of surface as far as possible, stereographic projection is the only available method, but even in this, areas near the edges of the map are four times too large.

To represent the surface unaltered in area, and to reduce the greatest angular distortion to a minimum, Lambert's equivalent azimuthal projection should be made use of.

Among the mean distance projections the equidistant projection (p. 79 *seq.*) gives the smallest angular and surface distortion.

External projections, in which the eye is placed at a distance from the surface less than the length of the diameter of the globe, give representations in which the greatest angular distortions are smaller than in Lambert's azimuthal projection, while the greatest surface distortions are smaller than in stereographic projection, but the distance distortions smaller than in either of these two, yet greater than in equidistant projection.

Conical projections give better results when the set form of the circumference is abandoned. We have hitherto been accustomed to imagine the cone as so placed that its axis coincides with the axis of the earth. But we may with equal right represent the eastern or western hemisphere on the conical surface, touching the earth along one of its meridians, the vertex of the cone coinciding with the extension of an equatorial radius of the earth.

If the object is to represent a considerable segment of the hemisphere, azimuthal and perspective projections are recommended.

TOPOGRAPHY

Classification of Maps

(a) *Names and General Classification of Maps*

The Greeks called their land-maps πίναξ, the Romans, *Orbis pictus*, fr which the old German word *Land-tafel* has been derived. The La *Charta* means originally record, letter, or information, but occurs as ea as the fourteenth century in reference to land-maps. The express *Mappa* dates from the ancient pictorial land-maps painted on canvas. H in England, we still distinguish *maps* (land-maps) and *charts* (sea-ma The name *Atlas* for a collection of maps owes its origin to Mercator's g cosmographical work; his heirs transferred the name, which origin belonged to the entire work, to a part of it, the collection of maps.

Maps are generally classified as *celestial*, *terrestrial*, and *nautical* ma

Among the celestial maps we reckon in the first place *astronomical n* representing the solar system, single planets, or the moon. Since lunar observations astronomical telescopes are required, which invert object, the south in lunar maps is at the top, the north at the bot Meridians and parallels appear on the disc of the moon the same as or earth, and the spherical co-ordinates of a point on the moon's surfac called *selenographic* or lunar co-ordinates; we thus speak of *selenogr* longitude and latitude.

Constellation charts are for the study of the starry heavens. T

give the constellations, with the separate stars clearly defined. The co-ordinates here used are right ascension and declination.

Land-maps are representations of portions of the earth's surface.

Sea-maps or *nautical charts* form an extensive group, in which not the conformation of the countries, but the coast-lines and the seas surrounding the land, are principally represented. The interior of the land is used in nautical charts for enlarged representations of important points, or for picturing nautical signs, buoys, beacons, and lighthouses.

Both land- and sea-maps can be subdivided into many different kinds, according as they are classified with regard to their scale of reduction, their contents and object, or their manner of construction.

Classification of Maps

(*b*) *According to their Scale of Reduction*

The scale of a map means the proportion in which all parts of the representation appear reduced in scale. If, for instance, two places on the globe are 1 kilometre apart from each other, and their distance on the map is 1 cm., then the scale is 1 : 100,000, which can also be expressed by the fraction $\frac{1}{100000}$. The scale is generally marked on the map. By means of the scale we can measure at once what part a certain line on the plan is of its corresponding line in nature. For instance, if we measure on the map a distance of 3 cm. (the scale being $\frac{1}{40000}$), this represents in nature a distance of $3 \times 40{,}000$ cm. $= 1200$ metres. But if the distance on the map measures 3 dm., this represents an actual distance of $3 \times 40{,}000$ dm. $= 12{,}000$ metres. Any figure, therefore, may be taken as unit of measurement. It is only of late years that this manner of scale drawing has been introduced.

The proportion of reduction is not always expressed in the same units of measurement. Sometimes it is stated what is the actual distance between two points in nature, their distance on the map being represented by a certain unit—for instance, 1 cm. Thus we may find on a map,

1 cm. = 500 m.; this means that every centimetre on the map equals 500 metres in nature. If the length of the course of a stream is found to be = 17 cm. on the map, this represents an actual length of 500 × 17 = 8500 metres = $8\frac{1}{2}$ kilometres. The method of measuring sometimes adopted on English maps is (1 inch = 15·78 miles, *i.e.* 1 : 1,000,000).

Maps are, moreover, generally provided with a scale on which distances are geometrically marked, and can be easily read off. On such scales the usual measures, either kilometres or miles, are given in their reduced form, say 1 cm. to 1 mile. The spaces marked on the scale give the representative distances of the distances as they actually are. Fig. 72 represents a rectilinear scale in proportion of 1 cm. = 1 mile. Each space

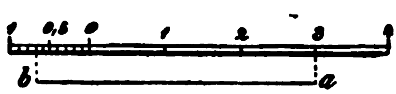


FIG. 72.

on it represents a mile, called a *reduced mile*. One of the spaces is sub divided into ten parts to represent tenths of miles. Thus if we want t determine the distance *ab*, we take the distance between the points of ou compasses, and mark it off on the scale from *a* to *b*. We then find tha the distance is 3·7, or, in other words, that the places are 3·7 miles distan from each other. This method of reduction is found on the old maps c the time of the so-called loxodromic maps until the last century.

In cases where no scale is given, we can easily ascertain the mea scale of reduction, by measuring the length of a degree of a meridian o the map, and dividing the same by the actual length of a degree of tl meridian. If, on the map, a degree of the meridian is 5 cm. long, the accepting the mean length of a degree of a meridian to be 111,121 m the proportional of reduction will be

5 cm. : 11,112,100 cm.
i.e. 1 cm. : 2,222,420 cm.

or in round figures, $\frac{1}{2222000}$.

This reduction, however, can only serve (especially when the projection is not equiangular) for the nearest vicinity to the place where the degree of meridian has been measured; for no distances are actually the same in any projection, and then distortions differ in different portions of the map, and in non-equi-angular projections they are even different for the same place in different directions. It is therefore not strictly sufficient for any map to have only one scale; but in practice it is found to answer every purpose to give the mean scale.

The scale of reduction is also influenced by the condition of the atmosphere, and depends upon the temperature, and particularly upon the moisture of the atmosphere, because the paper on which the map is drawn contracts in cold and damp climates. This is, however, small.

Most European states measure now by the metric or decimal system, with the metre as unit of measurement. A metre was intended to be the ten-millionth part of a quadrant of the earth, and is so very nearly.

In measuring large tracts a mile-scale is used. A nautical mile or knot [1] is the length of one minute of a great circle on the globe, and is the same as a British geographical mile. Thus:

1 nautical mile = 1855 metres, roughly.

Maps have sometimes, moreover, the following measurements marked on them:—

1 Austrian post mile = 7586 m.
1 Prussian mile = 7532 m.
1 English statute mile = 1609 m. = 1760 yards.
1 Russian (verst) = 1066 m.

The surface size of countries is given in square miles or square metres, generally in square kilometres.

The scale of reduction depends upon the kind of map that is to be constructed. We observe the following divisions:—

1. *Plans* and *field maps*; scale varying from 1 : 500 to 1 : 10,000 for

[1] The *knot* is more correctly the unit of *velocity* in navigation, *i.e.* a *velocity* of one mile (nautical) per hour, but the term is often inaccurately used for a *distance*.

official maps, technical plans for river regulations, road and railway construction, etc. Scales varying from 1 : 2000 to 1 : 5000 are the most generally used.

2. *Topographical special maps*; scale 1 : 10,000 to 1 : 200,000.

3. *Geographical maps*; *survey maps*; scale 1 : 200,000 to the very smallest scale of reduction.

This scale classification, however, must not be taken too strictly, because a map constructed on a medium scale may be either a general or a special map, according to its contents.

In sea-maps (charts) the following divisions have to be noted:—

1. *Coast* or *special maps* (scales varying from 1 : 10,000 to 1 : 30,000), used in sailing along the coast, through straits, and the entrances of bays, river-mouths, and harbours.

2. *Sailing* or *course maps*, for ordinary use during the voyage, more especially indicating the geographical position and the course. The scale for these charts should be so constructed that minutes can easily be read off on both scales for latitude as for longitude.

3. *General* or *survey charts*, for taking general bearings on long ocean voyages. The scale varies from 1 : 800,000 to 1 : 1,000,000.

Classification of Maps

(c) *According to their Destination*

Modern cartography enters so minutely into all details of figurative representation that it is somewhat difficult to bring all cartographical productions, as far as their object or contents are concerned, under some definite head. We therefore confine ourselves to specifying the principal groups.

1. *Geographical maps* in general purport to give a faithful representation of the earth's surface or a portion of it, with all the details necessary to impart a general knowledge of the area, or to serve some special object of orientation, as far as this can be done within the narrow limits of the reduced proportion.

2. *General physical maps* have for their object to represent the general physical conditions, or the particular physical condition of a given area, to the neglect of all other details foreign to this one special object. Under this head we distinguish :—

(*a*) *Geognostic* and *geological maps*, illustrating the structure of the earth's crust, the different rocks of which it is composed, and their bearing upon the formation of the periods in the history of the earth.

(*b*) *Hydrographical maps* represent waters of all kinds—rivers, streams, brooks, canals, lakes, ponds, etc.—stating the velocity and navigableness of the respective waterways, depths of lakes, etc., giving the currents, bridges, and ferries.

(*c*) *Orographical* or *mountain maps* are chiefly intended to represent the inequalities of the soil. They show the distribution of mountains and their bearings, and give heights, and positions of saddles, ridges, and passes.

3. *General biological maps* view the earth as the abode of human, animal, and vegetable life. According as they depict the distribution of the different families of mankind, of the animal or of the vegetable kingdom, they are distinguished as *ethnographical*, *zoological*, or *botanical* maps. The first-named form the more numerous and important subdivision; they show the distribution of mankind into groups and races, the various languages, nations, manners and customs, religious views, speech, diseases, etc.

4. *Political maps* depict the distribution of administrative power over the earth, and its different governments. If they treat of the conditions of past periods, they are called *historical maps*.

5. *Commercial* or *route maps* are those representing the natural and artificial routes and means of transport for communication and commerce. On the *general route maps* all the means of communication of certain tracts of land or of states are given in broad, though accurate, outline, to the neglect of the smaller details. Special route maps, such as railway, road, telegraph, postal, and navigation maps, give all details, such as stations and stopping-places, distances, milestones, stations for relays of horses, changing-places, etc. The navigation maps of more modern construction

(for instance, Chatelain's map of the world) show the flags of steamers of the different lines, and also mention how many voyages are made per month or per week. To this class of map belong the recently introduced *isochronical maps*, specially important for economical necessities. They show which places can be reached within a certain time from some given centre. *Nautical route maps*, also a special kind of route map, give in differently coloured lines the tracks of vessels suitable for different seasons of the year.

6. *Statistical maps* illustrate the individual distribution of the human race (maps of density of population), also the conditions under which the people live from the point of view of political economy, their occupations, industries, trade, and agriculture (economico-geographical maps).

7. The series of *special physical maps*, treating more particularly of the earth's atmospheric phenomena, is too extensive to enumerate. Amongst the more important we mention *magnetic maps*, showing the distributior of magnetism over the earth; *meteorological maps*, representing the distri bution of temperature, moisture, and motion in the atmosphere; *clima tological maps*, showing the distribution of climatic differences resultin from the combination of these various factors; *oceanological maps*, illus trating the relative temperature, gravity, and motion in the differen oceanic regions, etc. The important group of *maritime charts* (*sea-maps* should also again be mentioned under this category.

In all maps which, like the last-named groups, serve some particula purpose, the special object to be illustrated is given in coloured patche or lines connecting the points of equal intensity of the phenomenon.

The maps here enumerated can finally be classified as *hand maps* c *school maps*. Hand maps are for the more advanced study of the subjec and for business purposes. School maps differ from hand maps, in th they are of more convenient size, and that their contents are suitabl limited and arranged, no matter whether they form part of a school atl for the scholar's use, or whether they are wall maps for general scho use. We also distinguish *hand atlases* and *school atlases*.

GRAPHIC REPRESENTATION OF THE CONDITION OF THE SOIL, ALTITUDES, ETC.

1. *Projection of Position*

The science of cartography has taught us several methods by which we can represent countries on a plane surface. According to one or another projection we construct a network, and mark on it the different points in their geographical longitudes and latitudes. These points may be towns, villages, or hamlets; to mark a river, a boundary line, a mountain chain, or a coast-line, we imagine the curved line under consideration cut up into a polygonal line of different lengths, in proportion to the scale of the map, and transfer the same on to the paper, according to the longitude and latitude of their extreme points.

Geographical maps in general are expected to give a correct idea of the nature of the soil, and to contain the objects of chief interest regarding habitability, cultivation, and communication. In general maps it may be necessary, both from want of space and for the sake of clearness and legibility, to omit many details, such as isolated houses, small brooks, land or forest paths, etc. In such cases the individual character is lost in the general type or characteristic representation at large.

This is especially the case in geographical maps reduced to more than one-millionth in scale. The original pattern is gradually superseded by a mere symbolisation of its topographical and geographical elements. Places of habitation are only indicated by special marks. Hamlets and small places are omitted altogether in thickly populated districts, also any less important roads, agricultural and other particulars, so that land-maps on a very small scale are merely abstract representations giving a general idea of the whole, its contours and its size.

Sometimes, if the object of the map requires it, certain small items of importance, which could not be brought within the scale of reduction, are represented disproportionately large. For instance, in a route map a first-class road would be a very important element, but the scale of the

map might not permit it to be sufficiently conspicuous if drawn in its true proportion; in such cases the object in question may be drawn out of proportion (indicating scale of enlargement), so as to draw attention to it.

Objects, the ground-plan of which cannot be faithfully represented, are indicated by certain signs, as much as possible in keeping with the original purpose. Such signs are called *signatures* or *conventional signs.* For the sake of clearness, such signs are inscribed in colours, figures, or special lettering. If colours are used, the following rules are usually observed:—

Water is washed out in dark blue, and sketched in in light blue. Objects of stone are red; wood or clay, black; pasture ground, heaths, meadows, gardens, bluish-green; woodlands, pale black; brushwood, yellowish-grey; vineyards, yellowish-red; rocks and boulders, reddish brown. Footpaths, border paths, and ordinary high roads, provided they form principal roads of communication, are given in chrome yellow Bogs and swamps are blue. Arable land, used as pasture land while lying fallow, or used alternately as such, remains white. The signatures ar the same throughout; they only differ in scale, and according to th special object of the map.

These simple signs have only lately come into general use, although to a certain extent, natural objects were symbolised on all ancient carto graphical productions. On the Roman route maps (*Itineraria picta* mountains were represented in the ordinary hill fashion, streams by thic curved lines, roads by thin straight lines. These latter were marked o in numbers, denoting the distance from place to place in stadia; th names of the roads were also inscribed along them. Large forests wer indicated by trees; towns and encampments by single houses. The olde specimen of this kind of map, being at the same time the chief monumen of ancient cartography preserved to us, is the so-called Pentinger Tabl (Fig. 73), discovered by Conrad Celtes in the beginning of the sixteent century, and then in the possession of the Pentinger family. This ma was first made known in 1591 by Wolfgang Welser, and republished i 1598 by Ortelius. The original drawing, on twelve sheets of parchmen

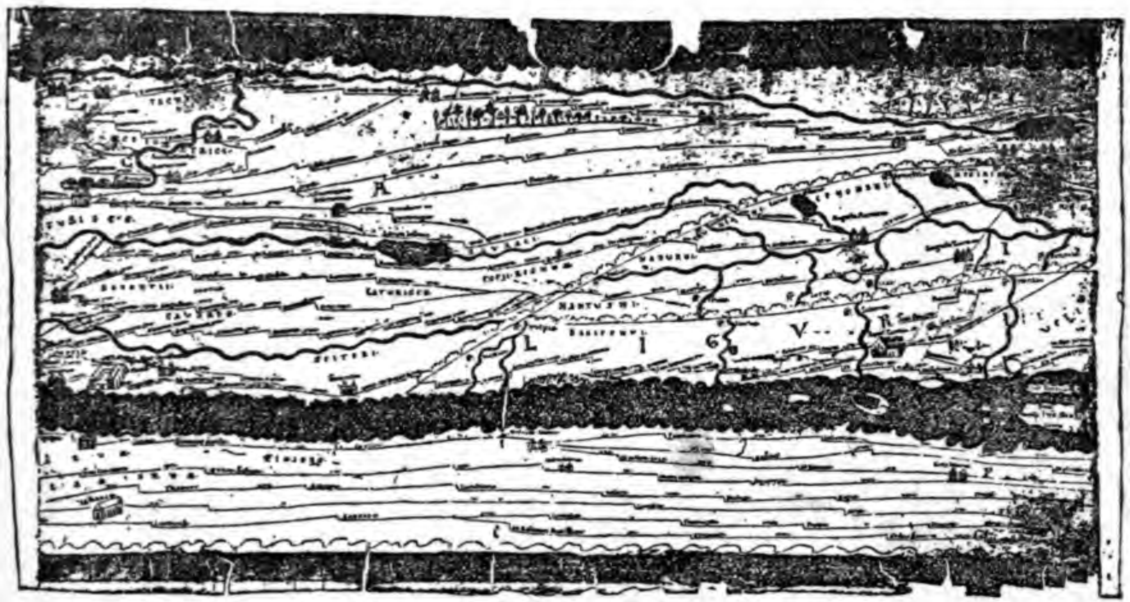


Fig. 73.—Pentinger Table.

now preserved in the Royal Library at Vienna, dates from the thirteent century, and is said to be constructed from third-century directions.

The world maps of the Middle Ages are very scantily furnished wit symbols. Their object was not so much to define the position of place as to determine and illustrate the general configuration. Hence the is a great lack of topographical detail, and all the more room for loc picture-designs of buildings, churches, historical or mythical event fabulous animals and monsters. The most interesting specimen of such world map is the so-called Erbstorfer map, constructed about 1270 i the neighbourhood of Lüneburg.

As late as the end of the sixteenth century, it was customary to decora maps in the most grotesque manner, especially the maps of Asia, Afric and America, of which continents but little reliable information was be had, and the empty spaces were filled in by anything the artist wish to give special prominence to. Thus towns, fortresses, and castles we represented by miniature towns, fortresses, and castles. Pictures of trees- for instance, three or four large palms or cocoa-nut trees—characterised t vegetation. Banners or flags in the middle of a country, or pictures sovereigns on their thrones, illustrated the reigning power. Hum figures, on a somewhat large scale, depicted the distribution of t different human races, and pictures of animals indicated the wonder fauna of the different lands. The map of Juan de la Cosa (1500) is ov loaded with such illustrations, as shown in Fig. 74, which represe Africa. Sebastian Martin, in his map of Africa (1544), distinguishes t different kingdoms rather by sceptres and crowns; we still find here a there trees with parrots in them, and in the neighbourhood of Cape Colo a large elephant; a one-eyed monster, placed near the Cameroons, is t representative of the Monoculi of the Middle Ages; but in spite of th remains of the old-world pictorial decorations, we can see in these maps the sixteenth century the dawn of the reform of cartography and of t modern style of topographic drawing.

The map of Bavaria by Philip Apian is the topographical masterpi of the sixteenth century; it is a woodcut in twenty-four plates (156

On this map, imperial cities, bishoprics, monasteries, towns, villages, as also the positions of looking-glass factories, glass-works, salt-works, mines, and medicinal springs, etc., are indicated by special signs; even the administrative and judicial districts are marked. Another excellent map of

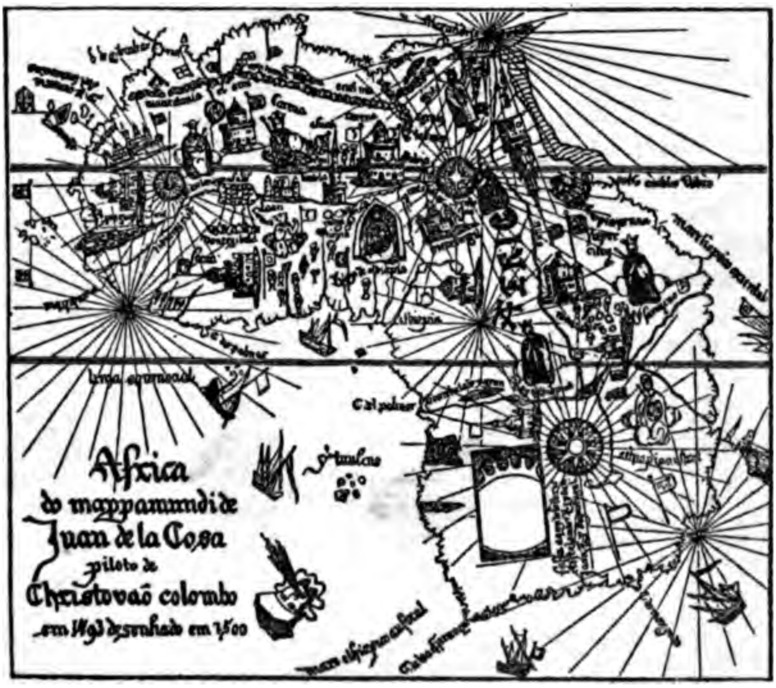


FIG. 74.—Mappa Mundi of Juan de la Cosa.

this description, a worthy counterpart of that of Apian, is the map of Prussia by C. Hennenberger (1576). On it the coast-lines and systems of irrigation are carefully noted; distinction is made between trees with foliage and firs; towns, fortresses, castles, villages, mills, etc., each have their distinctive mark.

Other topographical geographers of this period are Matthew Oede A. F. Zürner, M. Helwig, Humphrey Lhuyds, etc. But the old meth of drastic representation still found favour, and was more especially us for constructing tourists' maps in bird's-eye view. A good specimen this style is the map of Meissen and Thuringia by Job Magdeburg. T whole map is covered with bright colours. Green woods, brown mou tains and rocks, blue water, and red roofs, all are seen in bird's-e view. Single mountains can almost be recognised by their shape.

This method of bird's-eye-view projection has come into fashion ag lately. It has the great advantage of representing not only the conto of a landscape, but also all its minor details as they really are. But is evident that it can only be applied to small areas, for those parts the landscape which lie at a distance from the point of observation beco necessarily disproportionately contracted. This difficulty can to so extent be removed by taking surveys from several different points view, and piecing these different projections together.

In the seventeenth century the improvements made in triangulation a brought topographical representation a considerable step forward. W the eighteenth century begins the period of geodetico-topographical la surveying, which has now become so universal that almost all Europ states possess good topographical maps.

2. *Unevenness of the Soil. Sea-level.*

So far we have only considered the projection of points with regard to their third dimension, height. All objects on the surface the earth, which fall within the range of representation on a map, h bulk, *i.e.* they have length, breadth, and height. The height elevation of an object cannot directly be indicated on a map, but we so approximately by marking in our projection the point where a plu line would touch the ground, if let down from the top of the eleva which we wish to determine.

When we say that the earth has the form of a globe, we do not

account of any elevations or depressions on its surface, because the height of the highest mountains is immeasurably small compared with the semi-diameter of the earth. Now although this is true enough, the fact also remains that we dwellers on the earth have constantly to reckon with the unevennesses of the soil; we cannot even take a walk without noticing whether the road is level or hilly. All heights are determined in relation to some level surface; on the globe, however, they are determined with regard to a spherical level of a certain semi-diameter, which must remain unaltered. It is, moreover, necessary that portions of this ideal globe should be actually visible and accessible from as many points as possible of the actual globe. All these requirements are met with only in the sea-level.

As the different parts of the ocean are connected, their surfaces, according to hydrostatic laws, should all be at the same level, or rather the sea should form an imaginary level globular surface round the earth. The sea, in fact, ought to have the same level everywhere; but this is not the case, because centrifugal force and the uneven distribution of solid mass over the surface of the earth disturb this proportion. We therefore agree to accept a *mean level.*

The ideal spherical surface, with regard to which all elevations are to be determined, is to be found in the mean sea-level. We therefore say the *absolute height* of a place or of a point is its vertical elevation above the mean level of the sea. But height is often determined with regard to some other level; for instance, the height of a mountain is measured in relation to the level ground upon which it rises. This is called its *relative height.* When several points have the same absolute height, we say that they lie on the same level.

We have seen that no projection of a sphere can be absolutely correct in its horizontal dimensions, and it is equally impossible to represent the true form of nature correctly, even on topographical maps of a sufficiently large scale to allow room for specifying the characteristic individuality of different heights. Just as the earth can only be correctly represented on a globe, to answer all requirements of conformity in area

and angle, so the model is the only representation which can claim to be absolutely correct in all its proportions.

The art of mountain-drawing has developed very slowly and gradually. In olden times people were satisfied to show the position of mountains on the map by so-called *indentations* (Fig. 75). In the first edition of Ptolemy's work, the high mountain ranges are thus indicated. Later on another method was resorted to, by which mountains were represented as rows of little hillocks. This method was in use till the beginning of the last century, when gradually other ways of representing heights were invented, so as to make their altitudes on the map more in conformity with their natural proportions.

Fig. 75.

Relief maps were first originated in Switzerland, where the mountainous nature of the ground seemed to make this form of representation almost an absolute necessity.

The first relief map was made in wax by General F. L. Pfyffer (1766-1785). Paste, plaster, and clay can also be used with advantage.

3. *Method of Horizontal Section-Lines*

We will assume that we have to represent in horizontal projection two cones, A' and B' (Fig. 76), which have equal bases, but are of different altitudes. These cones we suppose to be mountains. The horizontal circumferences of the two bodies will be two circles A and B, with equal radii; but we cannot learn from this projection which is the higher of the two. Suppose, however, we mark off each cone in equidistant sections $m'n' = m''n''$, we shall find that the higher cone contains more sections than the lower one, in our case A' has seven sections and B' only four. We further imagine lines drawn through the points of division and parallel to the plane of the base, thus cutting the conical surface into circular planes. The horizontal projection of these circles will be found with the help of the projecting vertical line a'_2a_2, a'_3a_3 . . . to form a

[redacted]

system of concentric circles, their common centre being at the projection of the top of the cone.

Now if we examine the relative horizontal projections A and B, we notice that the higher cone has more concentric circles than the lower one, and as they are distributed over areas of the same size, the circles in the case of the higher cone lie closer together than in the other.

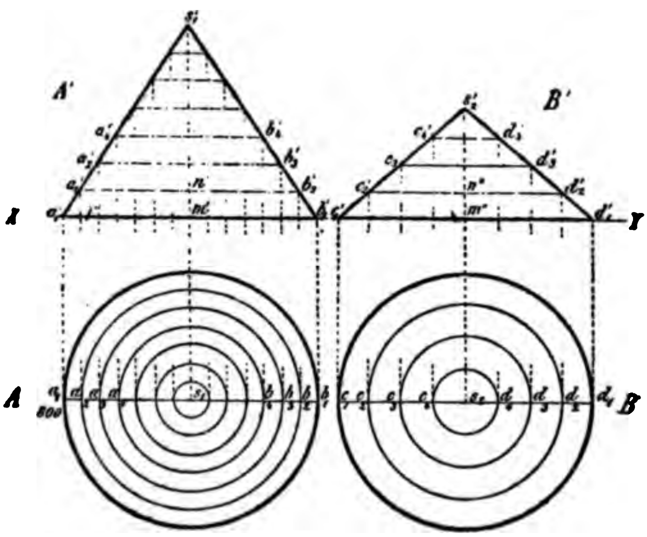


Fig. 76.

The first principle, therefore, in determining heights is this: the greater the number of concentric circles, and the closer they lie together, the greater must be the elevation of the body. We can go further than this, and contrive to read off the elevation of the two given bodies above the plane directly from the horizontal projection. We assume that $m'n'$ equals 5 metres (actual length); we count the concentric circles, and thus easily determine the height of any given point above the plane XY. In our cone AA' we count seven concentric circles — for

the point representing the projection of the vertex may also be reckoned as a circle; hence the height of S_1 equals 5×7 m. = 35 m. And if the elevation of the base A has been determined—is, for instance, 800 metres — then the absolute height of the point $S_1 = 800$ m. + 35 m. = 835; in other words, S_1 lies 835 m. above the sea-level.

Upon this principle rests the method of land-surveying by means of horizontal section-lines. In horizontal section projection, also known as *section projection*, we imagine a given body on the plane to be intersected by a certain number of equidistant horizontal planes. On the surface of this body we determine the boundary lines of the horizontal planes, and thus obtain a series of lines, called indifferently *horizontals*, *horizontal lines of intersection*, *curves of level*, or simply section-lines. Each portion of the body thus contained between two horizontal planes is called a *section*; for instance, $a'_2 a'_1 b'_1 b'_2$ (Fig. 76) is a section. The perpendicular distance between two horizontal planes, as $m'n = a'_2 f$, is called the *elevation* of a section. The side surfaces of the sections, or the conical configuration, are called the *faces* or *inclines*. The area contained between two neighbouring lines in horizontal projection is called the *belt* of intersection.

In Fig. 76 the vertical plane $a'_1 s'_1 b'_1$ passing through the axis of the cone represents the vertical projection of a section of our imaginary mountain. This is called a *profile*. To represent the horizontal projection of a circular body, one profile suffices. But a cone might be of different structure at the back, *i.e.* at the side not visible on the plan; in that case A would be no longer the true horizontal projection of A'. Seen from another side, the body in question might look as in Fig. 77. The upper diagram would then represent the profile of A', assuming a plane to pass through $s'_1 m'$ perpendicular to the vertical plane. When we divide this body into sections, and project these on to the horizontal plane, we get a totally different figure. One profile, therefore, is not sufficient to represent the true shape of an object. When dealing with an irregularly shaped body, we have to take several profiles, and represent the horizontal projections of all of them. If, for instance, the unevenness ABC (Fig. 78)

[redacted]

is given, the first thing to do is to draw and project horizontally the section-lines $m'p'$, $n'q'$, $o'r'$. The horizontal projection of the profile $A'B'C'$ is the line AC. Projecting points m', n', o', p', q', r', we get points

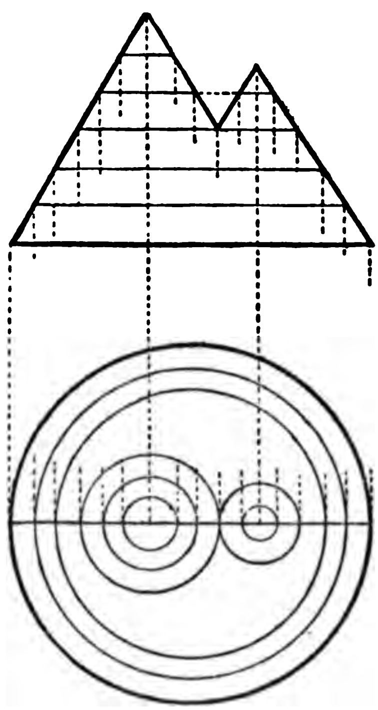


FIG. 77.

m, n, o, p, q, r; these points, however, are not sufficient in themselves to describe the curves of level; other profiles are required besides. We imagine a plane passing through $B'D'$, perpendicular to the vertical plane (or elevation); the horizontal projection (plan) of this profile will be D_1D_2. Let $D'_1B''D'_2$ be the profile (Fig. 78). We project the points of inter-

section of this profile on the horizontal plane by $a'_1 a'_2$, $b'_1 b'_2$, $c'_1 c'_2$, and find in the horizontal projection on the line $D_1 D_2$ the corresponding points a_1, b_1, c_1, c_2, b_2, a_2. We take another profile, its horizontal projec-

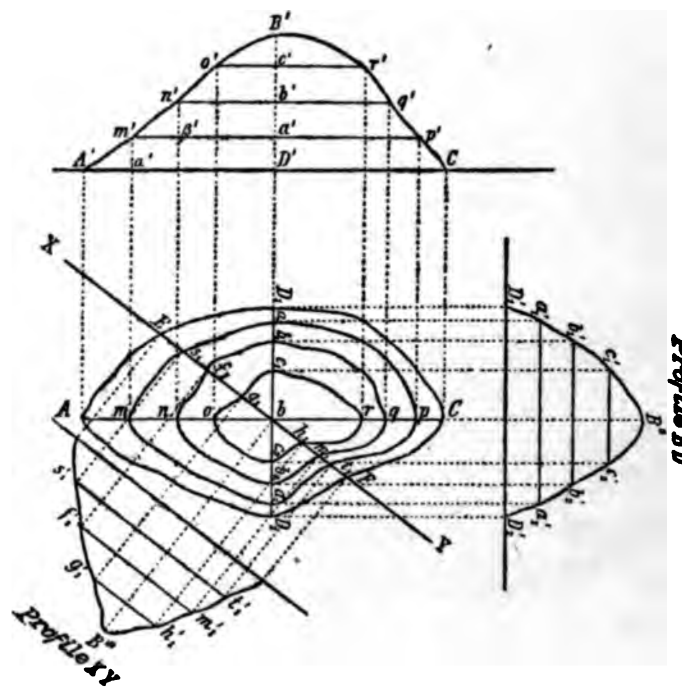


FIG. 78.

tion being EF, and obtain in like manner points s_1, f_1, g_1, h_1, m_1, t_1. It is necessary to draw and project as many profiles as we need points in the horizontal projection, which, being joined together, will suffice to represent the elevations in different points and in different positions, or, in other words, until the curves of equal level have been sufficiently determined.

The small triangles $A'm'a'$, $m'n'\beta'$, etc., are called profile triangles; $m'a'$ or $n'\beta'$ is the section elevation; $A'a'$ or $m'\beta'$ is the base-line; $A'm'$ or $m'n'$ is the slope, and $\angle a'A'm'$ or $\beta'm'n'$ is the angle of inclination. Given two sides of a profile triangle, the third can be found by calculation or construction.

The construction of the curves of equal level is quite simple in practice. In land-surveying we determine not only the opposite position of points with regard to the horizontal, but we also measure the elevation above sea-level of some prominent objects, in order to determine the altitude of other points in relation to these. Given the difference of elevation of these points, and the absolute height of a few standard points, the absolute height of all points can be determined, and the figures thus obtained can be transferred on to our plan. By connecting all points of the same altitude, we obtain the section-lines.

In measuring altitudes the choice of stations is of the utmost importance. On evenly inclined (sloping) ground it is sufficient to determine the altitude of three points not lying in a straight line, because a plane is determined by three points not lying in a straight line. But when the inclination (slope) is variable, more measurements are required. The line in which an evenly sloping plane intersects a plane of variable slope is called the fault-line. By this fault-line it is necessary to take a greater number of measurements. Besides the fault-lines, there are the *lines of greatest inclination* towards the horizontal plane. These are also of importance. They are determined by the direction of running water. Where the ground curves evenly, and no regular fault-lines occur, points of elevation have to be determined along the line of greatest inclination.

If we simply connected the points of equal elevation by section-lines, the corresponding absolute height of every section-line would have to be inscribed on it, and this should be avoided. Instead of this we mark the absolute height of a few points, and note the section-lines for certain different sea-levels, *i.e.* for certain elevations. For instance, beginning at the top of some given mountain, we first connect by horizontals all points at 10 metres lower down, then those at 20, 30, etc. Then let n be the

number of section-lines between two stations, h the sectional elevation, H the absolute height of the one, H_1 the absolute height of the othe point, we get

$$H_1 = H + nh \text{ and so } h = \frac{H_1 - H}{n}.$$

Example 1. Supposing we are at station A, which is 540 metres high and want to go to B. Between A and B there are twelve horizonta (section-lines); the difference of level for each section line is 10 metres What is the height of B?

Answer. $540 + 12 \times 10 = 660$ metres.

Example 2. Between A and B we count twelve section-lines. A point A on the map we find marked 540 metres, at B, 660 metres. Wha is the sectional elevation of the map?

Answer. $$h = \frac{660 - 540}{12} = 10 \text{ metres.}$$

As a rule, the sectional elevation is marked on the map; if not, it i easily determined by applying Example 2, by counting the section-line between two given points, whose absolute elevation is known.

To determine the height of a point which does not lie directly on section-line, we draw a line of greatest inclination through this poin This line must cut the two nearest section-lines at righ angles. We then estimate the distance of the point i question from the nearest section-line in segments c sectional elevation, and add the segment of section elevation to the height of the lower section-line. Fo instance, if we want to know the height of the point (Fig. 79), situated between the horizontals 300 and 32 we draw the line zu perpendicular to these two section-lines, and fin that zx is about $= \frac{1}{4} zu$. Now as $zu = 20$, the required height of x is

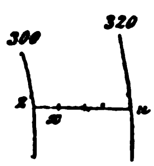


FIG. 79.

$$300 + \tfrac{1}{4} \times 20 = 5 = 305 \text{ metres.}$$

[redacted]

It is often necessary to know the angle of inclination (dip), *i.e.* the degree of the slope of the ground towards the horizon. We take the triangle of inclination $A'm'a'$ (Fig. 78), in which, because of the smallness of the area, the line $A'm'$ may be supposed to be a straight line, and find that

$$\tan m'A'a' = \frac{m'a'}{A'a'} = \frac{\text{sectional elevation}}{\text{section plane}}.$$

This gives $m'A'a'$ the angle of inclination.

Let in Fig. 80 ad be the distance of the section-line on the map, and dc the sectional elevation; it follows that

$$\tan dac = \frac{cd}{ad} = \frac{\text{sectional elevation}}{\text{section plane}},$$

and $\angle dac$ is the angle of inclination.

Whenever we find marked on a plan that the distance of the section-line is equal to the space ad, we know that the ground at that point inclines at the angle dac. The scale (Fig. 80) for measuring distances is based upon this fact. To construct the scale of inclination we draw a straight line ad, and with the help of a protractor we draw the other lines marked respectively 5, 10, 15, . . . up to 65, in such a manner that

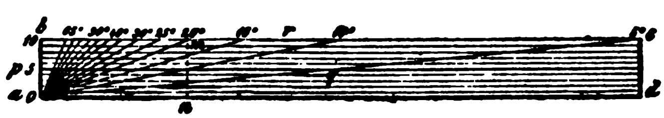


FIG. 80.

they are contained within the respective angles with ad of 5°, 10°, 15° . . . In point a we draw the line ab perpendicular to ad, and make ab equal to the sectional elevation of the scale of the plan. For instance, if the sectional elevation is 20 m., and the scale of the plan is 1 : 40,000, ab must be made equal to $\frac{1}{2}$ millimetre, for 20 metres = 20,000 millimetres.

From the point b we draw bc parallel to ad. The distances of the points

of intersection 5, 10, 15, . . . on b mark the distances between the consecutive surfaces of level, corresponding with the angles of inclination 5°, 10°, 15° respectively. For instance, take any distance bm:

$$\tan man = \frac{mn}{an} = \frac{\text{sectional elevation}}{\text{section plane}};$$

hence $\angle man$ = angle of inclination.

To determine the inclination of any given point on the map, you measure (with the compasses) the distance between the two nearest section-lines. One point of the compasses is placed on b of the scale, anc at the place where the other point touches the line bc we read off th inclination. Should the required distance not coincide with any of th distances set off in the scale, its angle is estimated as nearly as possible.

Very small distances are difficult to work with; we therefore tak for ab a larger figure—for instance, 10 mm., as in our diagram. W divide ab into ten equal parts, and through the points of division we dra lines parallel to the line ad. The triangles apq and abc being equal, w find that

$$ab : aq = bc : pq.$$

Hence if $ab = n \times ap$, bc must also be $= n \times pq$. Should the proportic of sectional elevation be too small, because of the scale of the map, w take the sectional distance between the compasses and place the compa points on the horizontal points of the scale, which correspond with tl respective multiples in the same proportion. This scale can be appli to any kind of map.

In the same manner as elevations on land are determined by mea of section-lines, the depths of the sea can also be measured, by connecti points of equal depth below the mean level of the sea. Lines of eq depth are called *isobathes*.

Isobathes are properly speaking the elder sisters in this family of curv for they were used before the *isohypses*, or lines of equal elevation, first the Dutch engineer Nicolas Samuel Cruquius (1678-1754), who in 17

[redacted]

drew the plan of the bed of the Merwede in lines of equal depth, and after him by Philip Buache, who represented the depth of the English Channel in isobathes. This chart was finished in 1737, and published in 1752 by the French Academy. On it the lines of equal depth are marked at intervals (10 fathoms). In the course of the following decade isohypses (lines of equal elevation) developed out of isobathes. The French engineer Millet de Mureau appears to have been the first who since 1748 marked elevations in his military plans. In 1749 he published a treatise in which he proposed to express the unevennesses of the ground by parallel lines with numbers of altitude. But it was not until 1771 that the French engineer du Carla laid a paper before the Academy of Sciences at Paris, accompanied by a plan of an imaginary island with horizontal lines, of which every tenth line was drawn thicker than the others. The editor of this treatise, Dupain-Triel, published in 1791 the first actual map of France containing the lines of equal elevation, with accompanying text and profiles.

The advantages of representing surfaces by section-lines are as follows. One can see at a glance (Fig. 81) if one point is higher than another, and how much; one also recognises at once points of equal height; and, moreover, it helps one to survey more easily the general structure of the territory. If the lines of equal elevation are fairly circular the rise is uniform on all sides. If the territory is a long-extended chain with a straight ridge of uniform heights, the ridge-line will appear in the drawing with parallel section-lines on either side of it, inclining towards it. If the section-lines curve towards the mountain, this indicates that there is a cavity at that point; if they curve away from the mountain, this indicates the existence of a headland at that spot. If the isohypses or lines of equal elevation are very close together, the ground is steep, and the steeper, the closer they lie together. If the section-lines are equidistant from one another, the inclination is constant, but if the distances increase from bottom to top, the inclination is convex; in the opposite case it is concave.

(a) Horizontal Section Lines

(b) Vertical Hachures.
After Lehmann After Müffling

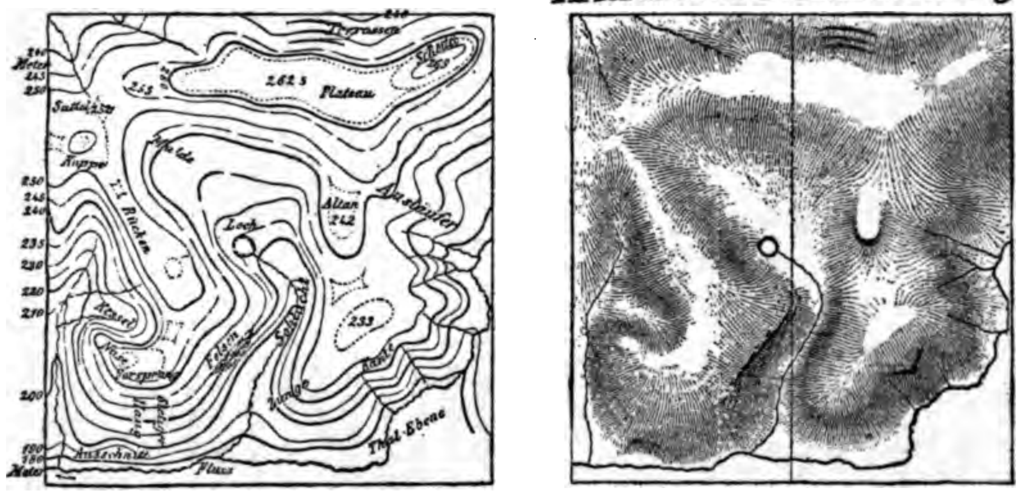


FIG. 81.

4. *To represent Comparative Heights by Colouring and Shading*

Maps with section-lines are convenient enough for reading off heights, but the process is rather elaborate, and it does not assist the appearance of the plan, nor does it help the surveying, when the ground has only a gentle slope. In order to gain these advantages, it has been proposed to represent the several altitudes by colouring or shading. Colouring is certainly the simpler method. The sectional elevations are coloured differently, and the tints are of increasing depth, whether they be shades of one colour, or entirely different colours. This method is applied with advantage in survey-plans on not too large a scale, say 1 : 100,000 to 1 : 1,000,000. The Austrian General Franz von Hauslab insisted that this principle should be followed, *the higher the darker*; whereby the darker shades remain confined to the small space of the high land, while the much-cultivated low land, which necessarily must contain more topographical information, remains clear and legible. Sydow favoured the opposite system; he began with the dark colours on the plane and graduated them up to white in the highest regions.

Another method to obtain clearness is the so-called washing operation. This method is based on the principle that the perpendicular rays of the sun send their full light upon the horizontal planes, but that sloping surfaces receive less light, in proportion to the angle of their inclination to the horizon. According to this principle, therefore, the portions of the map indicating a greater angle of inclination are kept darker. The washing process itself is the putting on with the brush of a brown or green tint, which is washed out towards the regions of smaller inclination and is intensified in steeper places. A well-drawn map of this kind is very effective, and brings out well the places of smaller or greater inclination. It is a favourite method for military and tourist plans.

In close connection with this method was the idea of expressing the configuration of the ground by so-called intermediate *isohypses* or horizontal lines, applied in the Norwegian official plans of scale

1 : 200,000. In this method the space between any two section-lines is divided, say, into ten parts, and through the points of division more section-lines — intermediate *isohypses*—are drawn; the higher the ground the closer these lines will lie together, because the spaces between the section-lines are smaller in proportion to the elevation of the ground. The effect of this method of representation is similar to the impression made by a coloured section-map, for as the intermediate section-lines lie so much closer together in the steeper places, they necessarily appear darker. This effect can be heightened by drawing the lines darker in the steeper places. But plans constructed after this method, besides having other disadvantages, can never be so clear and legible as the other methods described, and they have therefore never found much favour.

The *vertical-line method* of J. G. Lehmann (1765-1811) is the one most generally adopted. In this method also the sun is supposed to stand right above the region under projection; the light therefore falls perpendicularly, and the inclined planes receive more or less light in proportion to their less or greater angle of inclination towards the horizon. The shades are introduced by lines drawn in the direction of the greatest dip (or greatest inclination) towards the horizontal plane, *i.e.* in the direction of running water; and there must always be a certain number of lines within a given space. The width of the lines and their accompanying spaces have a special proportion for every inclination. By weighing these different proportions the topographer is enabled to express the angle of inclination of any given surface, and the map-reader finds it by estimation. Ground-plans of more than 45° inclination are seldom represented by this method, because such angles do not often occur, and then only in rocks, which are as a rule impassable and useless for military or agricultural operations. The shading process introduced by Lehmann demands that the proportion of white and black over a certain space shall be, for n degrees of inclination, as $(45 - n) : n$. For the sake of clearness he accepted nine gradations of shade, for angles of 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°. For instance,

if the inclination is 30°, the white space (W) must be in proportion to the linear width (L) as:

$$W : L = (45 - 30) : 30 = 15 : 30 = 1 : 2.$$

Over a space of 9 mm. the linear width therefore should be 6 mm.

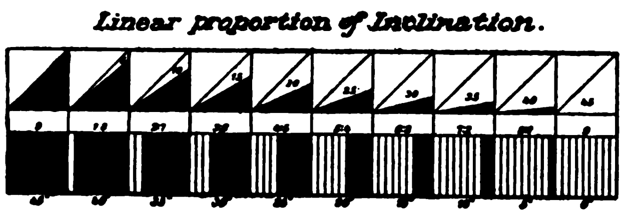


FIG. 82.—Scale of Slope.

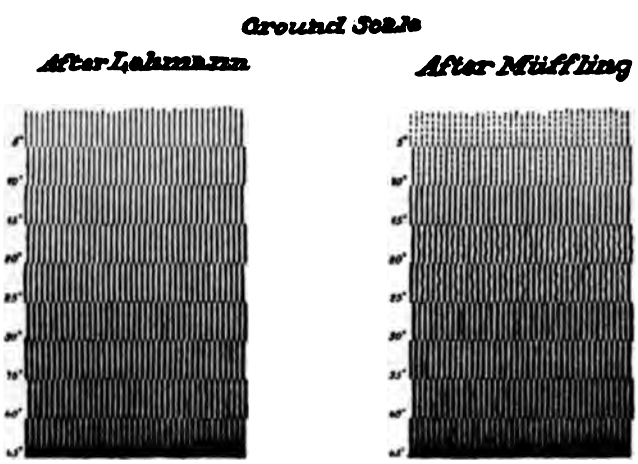


FIG. 83.

and the white space between 3 mm. Adapting the above formula for the proportion of all inclinations, we construct the following scale:—

Angle of Incl. to hor.	W : L	Angle of Incl. to hor.	W : L
5°	8 : 1	30°	3 : 6
10°	7 : 2	35°	2 : 7
15°	6 : 3	40°	1 : 8
20°	5 : 4	45°	0 : 9
25°	4 : 5		

According to these figures, for inclinations rising by 5 degrees the linear width increases one space each time, and the white between decreases one space. The numbers of the scale can easily be remembered when we keep in mind that the sum of linear width and white space between must always be 9. Linear width equals the fifth part of the numbers representing the degree of inclination, and the white space equals the difference between the quotient and 9. For instance, to find the proportion for 15 degrees inclination, we say $\frac{15}{5} = 3$; the number 3 indicates the number of spaces to be occupied by linear width, and the difference, $9 - 3 = 6$, shows the number of spaces that must be left white.

The *linear scale* is the A B C for the reading off of maps.

Many attempts have been made to improve Lehmann's system, with a view to simplifying the surveying of territory for military purposes.

In 1821 General Müffling introduced dotted, wavy, and alternating thick and thin lines (Figs. 81 *b* and 83). The general staff map of Germany, scale 1 : 100,000, is based upon the combined scales of Lehmann and Müffling. For level districts one degree has been added for 1° inclination. Bavaria and Austro-Hungary, with a view to their high mountain districts, have their scales augmented respectively by 60° and 80°.

For Bavaria the scale is determined by the formula $W : L = (60 - n) : n$, 5 being deducted from all figures arrived at. For Austro-Hungary the formula is: $W : L = [80 - (n + 3)] : (n + 3)$. Upon these two formulæ the following scales are constructed:—

	For Bavaria. W : L	For Austria. W : L
5°	11 : 1	72 : 8
10°	10 : 2	67 : 13
15°	9 : 3	62 : 18
20°	8 : 4	57 : 23

	For Bavaria. W : L	For Austria. W : L
25°	7 : 5	52 : 28
30°	6 : 6	47 : 33
35°	5 : 7	42 : 38
40°	4 : 8	37 : 43
45°	3 : 9	32 : 48
50°	2 : 10	27 : 53
55°	1 : 11	22 : 58
60°	0 : 12	17 : 63
65°	...	12 : 68
70°	...	7 : 73
75°	...	2 : 78
77°	...	0 : 80

Müffling proposed to construct all scales upon the decimal system, but this idea has not yet been carried out.

So far we have only dealt with the relative proportion of linear and blank spaces. We must now determine the absolute width of the linear space. The underlying principle is, that the smaller the scale of the map, the smaller the width of linear and blank spaces, and consequently the greater the number of lines to the centimetre.

Linear length is determined by the angle of inclination; the smaller the angle, the longer the length. Let in Fig. 84, *a*, *b*, *c*, *d*, *e* be the section-lines; the angle of inclination for the distance *ab* marked on the scale of inclination must be equal to the angle at which the linear width has been entered. Hence we have the following rule: Linears must reach from one to another of the imaginary section-lines drawn on the plan.

FIG. 84.

Therefore, to draw these lines on a map, the section-lines must first be drawn in pencil, then the linears are put in stretching from line to line, and when these are all entered the section-lines are rubbed out. The linear method is thus based on the section system, because the linears must stand in the direction of greatest inclination towards the horizontal plane, and therefore be perpendicular to the section-lines.

We can only shortly refer to the linear method applied to oblique

illumination as practised formerly, especially in France and Italy. The Lehmann principle is followed, but the rays of the sun are supposed not to come down perpendicularly but at an angle of 45° north-west inclination, so that the length of a shadow depends not merely on the inclination, but also on the bearings of the territory under projection. As, however, it is generally desirable that horizontal planes should remain white, but that slopes inclining towards the north-west at 45° should not be left quite white on the map, this method involved so many exceptions, more or less dependent upon the discretion of the designer, that the correctness of the projection became somewhat questionable. This method was therefore abandoned, until Dufour once again employed it for his map of Switzerland, scale 1 : 100,000; the most perfect piece of cartographical work in existence. It must, however, not be forgotten that the effect of this masterpiece has been obtained at the cost of two serious errors. The mountain slopes facing south and east appear steeper than those facing north and west, and the naturally sunny slopes with southern and eastern aspect present a gloomy, dismal appearance, while the really shady northern and western slopes appear clear and bright. Of late years (1878) the linear method with oblique illumination has been thoroughly and mathematically examined by H. Wiechel, and there is a future before it, for in connection with section-lines it is the best method for Alpine orography.

5. *Combination of Section-Lines and the Linear Method*

Section maps have the advantage of being easily read, and the configurations of the land are easily recognisable when the section-lines are not too far apart; but they have the disadvantage that when the section-lines are far apart the representation of the land becomes a much more serious matter, and that even certain configurations and modifications cannot be clearly brought out. For the representation of entire countries the disadvantages of the section system are even more prominent.

On the other hand, the careful execution and reading of lineated maps

is not so easy. For if it is difficult for the designer to maintain the exact proportion of linear and intermediate blank spaces, with reference to the different angles of inclination, it must be for the map-user still more difficult to recognise this proportion and the exact inclination of the land. But the linear system has this advantage, that it can be applied no matter what is the scale of reduction, and that it expresses the smallest forms compatible with the scale of reduction with great clearness, and altogether produces an intelligible picture.

Considering the pros and cons of these two methods, it has been thought advisable to combine the two, and hence we have the combined method of configuration. It consists in entering both section-lines and lineation (Fig. 81). The former make the reading easy, the latter produce a more correct representation. Linears and horizontals determine their relative position themselves, because they can only intersect each other at right angles. When entering the linears they should not be extended, as on the left side of Fig. 81, from one section-line to another, as with the increasing divergence of the linears the (proportion) relation between the linear widths and blank spaces alters, and the lineation would therefore be incorrect. Intermediate section-lines have therefore to be introduced, and between these the vertical linears corresponding to their respective inclinations are entered, as shown on the right side of Fig. 81.

This method is objected to because it fills up the maps so, and section-lines are often mistaken for lines of cultivation or communication. In spite of this drawback, surveying is materially assisted by the introduction of principal, intermediate, and auxiliary section-lines. The principal section-lines serve to mark (elevations) differences of altitude of from 50 to 100 metres; the intermediate section-lines denote differences of altitude of from 10 to 20 metres, and the auxiliary section lines refer to equidistances of from 5 to 10 metres. The first are drawn thick and dark, the next finer, and the third very fine. The intermediate and auxiliary section-lines, moreover, should only be drawn when likely to be legible.

Fig. 81*a* gives a representation based on this principle. The vertical distances of the principal section-lines are always 20 metres, divided by

intermediate section-lines of 10 metres apart (the former darker than the latter). A further subdivision can be made by introducing normal section-lines of 5 metres apart (broken lines), or if necessary auxiliary section-lines are entered of 2·5, or even 1·25, metres distance apart where necessary.

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